

## Report on the activities under the Short Term Mobility project *Transversely Isotropic Masonry-Like Materials*

Briefly speaking, the goal of the project was to continue the study of the constitutive equation of anisotropic masonry-like materials, especially the differentiability of the stress function.

The performed work is divided into two parts.

*The first part* consists of the examination of the differentiability of the orthogonal projection onto a general closed convex set under realistic assumptions on the smoothness of the boundary of the convex set. The existing literature establishes the continuous differentiability of the projection under the assumption that the boundary is twice continuously differentiable; an assumption that excludes the application to masonry-like materials (as explained in more detail below). Typically, the boundary of a convex set has faces, edges, and vertices (and analogous features in higher dimensional spaces), as established in a general result of G. Alberti (1994). A theorem was proved that under a slightly strengthened form of this assumption, the projection is piecewise continuously differentiable on a finite number of regions which correspond to the faces, edges, and vertices mentioned above. A preliminary version of a paper entitled *Differentiability of the metric projection onto a convex set in  $\mathbb{R}^n$  with the boundary singular points* was completed and is currently prepared for publication in a suitable mathematical journal.

*The second part* of the project applies the results of the first part to masonry-like materials. This requires to examine the boundary of the convex cone of negative semidefinite stresses that occur in the masonry-like materials. It turns out that the boundary of this set is not twice continuously differentiable, but has the structure outlined in the first part of this report. Namely, it consists of the face which is an infinitely differentiable manifold formed by negative semidefinite tensors of rank 2, of the edge of negative semidefinite tensors of rank 1, and of the vertex at the origin. Corresponding to this, the stress function of a masonry material of arbitrary symmetry will be subject to four regimes in the space of strains, in each of which it is infinitely differentiable. A publication is prepared, entitled *The differentiability of the stress-strain relation in no-tension materials*, to be published in a mechanical journal to be determined.

The described results give some hope that it is possible to determine the stress function of a transversely isotropic no-tension material and its derivative explicitly enough to be suitable for a numerical implementation in the finite element code NOSA-ITACA for the static analysis of masonry constructions. A further collaboration on the subject is planned.