SHORT TERM MOBILITY – 2009

Stay of Prof. M. Chipot at IAC-CNR in the period 3-16 September 2009

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***Title of the program*: VARIATIONAL ANALYSIS OF MAGNETOELASTIC LAMINATES**
(Analisi variazionale di laminati magnetoelastici)

Report

Smart materials (such as piezoelectric or magnetostrictive materials, shape memory alloys,…) are new materials which allow to realize an active control of elastic structures. The topic of interest here is the theoretical study of mathematical models which describe the interactions between elastic and magnetic processes. In our modelling the starting point is the equation of spin fields in ferromagnets, introduced by Landau-Lifschitz, describing the dynamics of micro-magnetic processes. The magnetization distribution, constant in modulus but not in direction, is well described by a functional composed by three terms namely the exchange energy, the elastic-magnetic energy and the elastic energy which accounts for the heterogeneity of the materials through variable (often discontinuous) coefficients.
The complexity of the problem requires the knowledge and the implementation of advanced techniques in calculus of variation and PDEs for the attainment of new theoretical results.

The activity carried out during the stay of Prof. M. Chipot at the IAC-CNR concerns the definition and study of a nonconvex energy functional arising in the interactions of magnetoelastic heterogeneous materials characterized by discontinuous elastic coefficients. In particular in one-dimensional case the energy functional can be expressed in terms of the only unknown function $\boldsymbol{m}$, which denotes the magnetization, and the scalar function σ which denotes the variable elastic coefficient. The proposed energy functional $\boldsymbol{F(m,\sigma)}$ is a Ginzburg-Landau functional with the addition of a nonlinear nonlocal term due to the missing explicit dependence of the functional from the elastic displacements.
The nonlocal nonlinear term is modulated by a positive parameter λ which characterizes the connection between the magnetic and elastic process.

The minimization of the energy functional is the aim of our program. In particular we prove that for λ small enough there are only trivial minimizers for the energy functional. Moreover there exists a

critical value λ^* such that for $\lambda > \lambda^*$ a branch of nontrivial global minimizers appears. For the study of this bifurcation problem and for the identification of the bifurcation parameter λ^* we have introduced and studied a new linear eigenvalue problem, depending on the elastic coefficient σ, that allows us to treat functions σ belonging to a large functional space.

We refer to the papers [2] and [3] (see references below) for the study of the static and dynamical problem in the case σ = constant. The new results obtained in the framework of this program will be detailed in the forthcoming paper [1], that appears as one of the first papers on the mathematical modelling of magnetoelastic laminates with degenerate and discontinuous coefficients. Among the papers and books on the subject of magnetoelasticity we also quote the references [4]-[7].

References

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