A Preliminary Estimate of the Stokes Dissipation of Wave Energy in the Global Ocean

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ABSTRACT

Wind generated surface gravity waves travel on top of a turbulent ocean. The turbulent Reynolds stresses in the upper layers of the ocean interact with the vertical shear of the Stokes drift velocity produced by the wave field to extract energy from the surface gravity waves. The resulting rate of dissipation of wind waves in the global oceans is about 2.53 TW on the average but can reach values as high as 3.7 TW, making Stokes dissipation as important as the dissipation of wave energy in the surf zones around the ocean margins. Unfortunately, this wave dissipation mechanism has hitherto been largely ignored. In this note, we present a preliminary estimate of the Stokes dissipation rate in the global oceans based on the results of the Wave Watch III model for the year 2007 to point out its potential importance. Seasonal and regional variations are also described.

1. Introduction

Surface gravity waves are frequently cited as a shining example of the very first successful application of the laws of fluid mechanics to a practical problem. However, for simplicity, most of the work over the past two centuries on oceanic surface gravity waves has considered the ocean to be inviscid. The interaction of waves with the turbulent motions in the upper layers has been ignored until recently. A proper treatment of the wave-mean current-turbulence interactions has only been accomplished in the past few years (Mellor 2003, Ardhuin et al. 2008, Rascle et al. 2008). The results show conclusively that very similar to the extraction of energy from mean currents by turbulence, turbulence can extract energy from the wave motions by the action of the

Reynolds stresses on the vertical shear of the wave-induced Stokes drift.

Wind generated surface gravity waves travel on top of a turbulent and *not* an inviscid ocean as is commonly assumed. This implies that there is an inevitable interaction between the turbulent motions in the upper layers of the ocean and these gravity waves. Overall, such an interaction leads to extraction of energy from waves by turbulence in the oceanic mixed layer. This interaction is particularly important for the dissipation of the low frequency part of the wave spectrum, swell (Kantha 2006). It acts as a source term for turbulent motions (Kantha and Clayson 2004) and a sink term for waves (Kantha 2006, Ardhuin and Jenkins 2006). This Stokes dissipation of wave energy is comparable to the dissipation of wave energy in the surf zones around the ocean basins, but the importance of this mechanism has not been fully appreciated and hence Stokes dissipation of waves has largely been ignored in wave modeling. In this note, we present a preliminary estimate of the Stokes dissipation in the global ocean based on the results of Wave Watch III model for 2007. Seasonal and regional variations are also described.

Needless to say that the Stokes mechanism of energy transfer from waves to turbulence constitutes an additional (in addition to the momentum and buoyancy fluxes at the air-sea interface) and important source of turbulent kinetic energy, and consequently, it enhances the intensity of turbulence in the oceanic mixed layer. One consequence of this is enhanced mixing and more uniform profiles in the mixed layer (McWilliams et al. 1997, Carniel et al. 2005).

2. Extraction of wave energy by turbulence in the mixed layer

Based on LES simulations of Langmuir cells in the ocean (McWilliams et al., 1997), Kantha and Clayson (2004) have parameterized the extraction of energy from surface gravity waves by turbulence in the oceanic mixed layer. They showed that the rate of change of turbulence kinetic energy (TKE) per unit mass can be written as (see also Kantha 2006):

$$\frac{d}{dt}\left(\frac{q^2}{2}\right) = -\overline{uw}\frac{\partial u_s}{\partial z} - \overline{vw}\frac{\partial v_s}{\partial z} \quad (1)$$

where $q^2/2$ is the TKE, $u_S(z)$ and $v_S(z)$ are the components of surface gravity wave-induced Stokes drift velocity, and $-\rho\overline{uw}$ and $-\rho\overline{vw}$ are components of the turbulent shear (Reynolds) stress; ρ is water density and z is the vertical coordinate positive upwards. It is the working of the Reynolds stress on the vertical shear of the Stokes drift that extracts energy from the wave motion and transfers it to turbulence. It is rather analogous to working of the Reynolds stress against the mean shear in converting kinetic energy of the mean currents into TKE.

The integration of Eq. (1) with z gives the rate of increase of the total TKE in the water column due to extraction of energy from wave motions. Because of the involvement of Stokes drift in this process, we can characterize this mechanism as Stokes production of TKE. This mechanism leads also to the dissipation of waves with the rate of dissipation of wave energy *E* given by:

$$\frac{dE}{dt} = -\int_{-\infty}^{0}\left(\vec{\tau}\cdot\frac{d\vec{V}_S}{dz}\right)dz \quad (2)$$

where $\vec{\tau}(z)$ is the shear stress vector and $\vec{V}_S(z)$ is the Stokes drift velocity vector. We call this the

Stokes dissipation of wave energy. To evaluate $\frac{dE}{dt}$, it is necessary to determine $\vec{\tau}(z)$ and hence it is necessary to appeal to a turbulence closure model (e.g., Kantha and Clayson, 1994, 2004) of the upper layers (see Kantha 2006).

Consider a monochromatic wave. The Stokes drift velocity can then be written as:

$$\vec{V}_S = \vec{V}_S(0)\exp(2kz) = c(ka)^2 \exp(2kz)\vec{k} = \frac{2\sigma k}{\rho g} E\exp(2kz)\vec{k} \qquad (3)$$

where k is the wave number, $\vec{k}$ is the unit vector in the direction of wave propagation, σ is the frequency, a is the wave amplitude, c is the phase speed and E is the wave energy; g is the gravitational acceleration. Therefore $\frac{d\vec{V}_S}{dz} = \frac{4\sigma k^2}{\rho g} E\exp(2kz)\vec{k}$ and Eq. (2) becomes:

$$\frac{dE}{dt} = -\int_{-\infty}^{0} \frac{4\sigma k^2}{\rho g} E\exp(2kz)\left(\vec{\tau}\cdot\vec{k}\right)dz \qquad (4)$$

For a general wave spectrum, Eq.(4) has to be integrated over the wavenumber spectrum to compute the rate of decay of the total wave energy in the wave field, or equivalently, Eq. (2) can be used provided the Stokes drift velocity is the sum of the contributions from the entire wave number spectrum.

Because of the exponential decay of Stokes drift velocity (and hence its shear) with depth, most of the interaction between the wave motions and turbulence takes place in the near surface layers. Since turbulence must be present for this interaction to take place, this also means that the interaction is confined to the depth of the active mixed layer. Therefore the Stokes depth, $1/2k$, is an important parameter. Because of the turning of the shear stress vector with depth, the latitude is also an important factor. Eq. (2) can be written as

$$\frac{dE}{dt} = -\alpha\left[\vec{\tau}(0)\cdot\vec{V}_S(0)\right] = -\alpha\left[\vec{\tau}_w\cdot\vec{V}_S(0)\right] \tag{5}$$

where $\vec{\tau}_w$ is the wind stress vector and $\vec{V}_S(0)$ is the Stokes drift velocity vector at the surface. Written this way, the constant of proportionality α becomes a function of the various factors involved in the wave-turbulence interaction. Its value is less than unity, with values decreasing somewhat with latitude because of the more pronounced turning of the shear stress vector with depth at higher latitudes, and generally increasing with Stokes depth. Kantha (2006) has explored the variability in α and has shown that its value is between 0.4 and 0.8, with higher values for higher frequency waves and lower latitudes. The use of Eq. (5) enables a rough estimate of the wave dissipation rate due to wave-turbulence interactions to be made without resorting to Eq. (2) and hence a second moment closure-based global mixed layer model. Simulations by Kantha (2006), using monochromatic waves and a second moment mixed layer model shows that the average value for the constant α lies is 0.33 for 15 sec waves, 0.58 for 10 sec waves and 0.87 for 5 sec waves. In this study we use a value of 0.65 for α appropriate to 8-10 s waves corresponding roughly to the peak of the wind wave spectrum. The estimates thus obtained for the wave dissipation rate are accurate to within a factor of about two. Our intention here is to provide a rough preliminary estimate of the Stokes dissipation rate to point out its importance. We postpone a more accurate estimate and necessarily a more extensive analysis to a later date.

3. Wave Watch III Model

As can be expected, observational data on the ocean surface wave field is inadequate to carry out this study. Instead appeal must be made to a global wave model. This is the same approach used by Rascle et al. (2008) to generate a global wave parameter database. Using a version of the popular Wave Watch III model, they show that on the average, the winds transfer energy into wave motions (which have a global energy content of about 1.52 EJ) at a rate of about 70 TW. Of this energy input, a majority of 67.6 TW goes into generating oceanic turbulence by breaking and other mechanisms, while 2.4 TW is dissipated in the surf zone. They did not however explore in detail the Stokes dissipation rate of the wave energy. Their value for Stokes dissipation of 6 TW, estimated as $\left|\vec{\tau}_w\right|\left|\vec{V}_S(0)\right|$ is perhaps an overestimate, since it does not allow for the possibility of nonzero angles between the two vectors. In this note, we intend to complement their work by computing the Stokes dissipation rate for the year 2007. Along the way, we also describe its temporal and regional variability.

To estimate the stokes drift over the global ocean, the WAVEWATCH III (WW3) Version 2.22 (Tolman, 1991; Tolman 2002a) was run at 1/2^{o} resolution (approximately 55 km at the equator) globally for the year 2007. This deep ocean wave model has been used at operational weather centers since 2000, and there has been numerous verification studies, using wave buoys and satellite observations, documenting its performance over the years (Rogers et al. 2005). It is a 3rd generation wave model, solving the non-linear wave Interactions using a discrete Interaction approximation (DIA). It uses the Tolman-Chalikov wind input formulation and dissipation terms that act separately on sea and swell. The waves are propagated using a 3rd order accurate finite difference method. The wave spectrum was discretized using 24 equally spaced directional bins and 25 logarithmically spaced wave number bins. The model was forced with 12-hourly surface winds from the Navy Operational Global Atmospheric Prediction System (NOGAPS), which have been archived on the Global Ocean Data Assimilation Experiment (GODAE) server. The time step was 3 hours. The Ice edge was updated every 12 hours. It is important to note that currently no wave model, including WW3 incorporates Stokes dissipation mechanism. Instead, the wave dissipation is parameterized by rather ad-hoc means with coefficients tuned to provide accurate estimates of the surface wave field for operational applications. For our purposes here, it does not matter how the wave dissipation is parameterized. What matters is that the resulting wave parameters be accurately reproduced by the model [For a detailed discussion of the skill of the WW3 model, see Rogers et al. (2005) and Rascle et al. (2008)].

The wave directional spectrum was integrated to estimate the Stokes drift velocity at the surface needed to compute the Stokes dissipation rate using Eq. (5):

$$\vec{V}_S(0) = \frac{(2\pi)^3}{g} \iint \vec{k}\, f^3 E(f,\theta)\, df\, d\theta \tag{6}$$

where f is the frequency (in Hz), $E(f,\theta)$ is the directional spectrum and θ is the propagation angle. Large and Pond (1981) formulation is used to convert the winds at 10 m to the surface wind stress. The wind stress and the Stokes drift velocity vectors at the surface are stored every 6 hours along with other wave parameters such as the significant wave height (SWH) and the peak period for further analysis. The energy in the wave motions is computed using the SWH.

4. Analysis and Interpretaion

The wave energy and the dissipation rate are summed up over the globe as well as the northern and southern hemispheres. In addition, values are computed for the longitudinal band 110^o to 285^o designated as the Pacific sector, 285^o to 25^o (Atlantic sector) and 25^o to 110^o (Indian sector). Figures 1a and b show (see also Table 1) the temporal variability of the energy in the global surface wave field and the Stokes dissipation rate of this wave energy. Black curves in Figures 1a and b denote the global values; the red curve in Fig. 1a corresponds to the northern and the blue curve to the southern hemisphere; the blue curve in Figure 1b corresponds to the Pacific, the red curve to the Atlantic and the green curve to the Indian sectors. The average value of global wave energy of 1.68 EJ (Table 1) is consistent with the 3-year average of 1.52 EJ estimated by Rascle et al. (2008). The significance of this value is evident from the fact that the total energy in barotropic ocean tides is only 0.6 EJ (Kantha and Clayson 2000b). The average Stokes dissipation rate of 2.53 TW is more than the 2.4 TW rate of dissipation of the wave energy in the surf zones at the ocean margins (Rascle et al. 2008), but can reach values as high as 3.7 TW. The importance of these values can be recognized by the fact that the dissipation rate of tidal energy in the global oceans and hence of the gravitational energy of the Earth-Moon-Sun system, is only about 3.75 TW (Kantha and Clayson 2000b).

Since the high frequency part of the wave spectrum contributes more to the surface Stokes drift values than the lower frequency (wave number) portions, as shown by Rascle et al. (2008, see their Figure 7), the above value is likely to be a slight underestimate due to issues related to the resolution of the high frequency end of the wave spectrum in wave models (The use of 24 frequency bands implies maximum frequency of 0.4 Hz instead of 0.7 Hz, which will reduce the surface Stokes drift value by a few tens of percent – personal communication by Dr. Fabrice Ardhuin). Because of the complicated dependence of the constant α in Eq. (5) on latitude and especially the wavenumber, this estimate of 2.53 TW average should be regarded as preliminary; more accurate estimates await the use of Eq. (2).

While the global energy and dissipation rates show very little seasonal variability, the hemispheric values show a prominent seasonal variability, with values in the northern hemisphere being higher during the boreal winter than the boreal summer but exactly opposite behavior is evident in the southern hemisphere.

Majority of the contribution to Stokes dissipation (1.79 TW) is from the southern hemisphere, with the roaring fifties contributing significantly, as can be expected because of the high wave energy there. During the boreal summer, the Stokes dissipation is mostly from the southern hemisphere. The average dissipation rate is 1.28 TW in the Pacific, 0.70 TW in the Atlantic and 0.55 TW in the Indian sectors. The Atlantic and Indian sectors contribute roughly equally to Stokes dissipation, with the Pacific sector contributing significantly more.

Figure 2 shows the distribution of the wave energy (in TJ) and Stokes dissipation rate (in MW) in each $1/2^o$ x $1/2^o$ box over the global oceans averaged over 2007. High dissipation rate regions are well correlated with high wave energy regions; for example the roaring fifties in the southern hemisphere, and the Gulf Stream and Kuroshio extension regions as well as the Arabian Sea in the northern hemisphere. The average dissipation rates reach as high as 70 MW and average wave energies as high as 36 TJ (in $1/2^o$ x $1/2^o$ box) in the southern hemisphere. The southern latitudes, the Gulf Stream and Kuroshio extension regions contribute heavily to Stokes dissipation. In the North Indian Ocean, the summer monsoon-affected regions along the Arabian coast display high wave energy and dissipation rates.

Figures 3a and b show the seasonal variability. Figure 3a shows the boreal winter (January-March) and Figure 3b the boreal summer (July – September) averages. The seasonal contrasts are rather striking. For example, during the boreal winter, high dissipation rates are evident in the northern hemisphere, while during the boreal summer, the values there are very low. In contrast, the values remain high in the roaring fifties region of the southern hemisphere during both boreal summer and winter, although the boreal summer values are significantly higher. In the North Indian Ocean, summer monsoon winds cause high dissipation rates in the Arabian Sea region.

The above findings are consistent with what could be expected based on what we know about the regional and seasonal characteristics of surface gravity waves in the global oceans. High wave regions tend also to be regions of high Stokes dissipation rates. There are no negative dissipation rate regions where the surface waves are extracting energy from turbulence in the upper layers. This is note worthy since it is different from the behavior of tidal working in the global oceans, where in some regions, the oceans transfer energy to the moon, while in most regions, the moon transfers energy to the oceans (see Kantha and Clayson 2000b).

It is worth reminding that what we have presented above are preliminary estimates using Eq. (5). While the trends and patterns are expected to be reproduced when more accurate estimates are made using Eq. (2) and a global mixed layer model, the numbers are likely to change somewhat.

Nevertheless, the study highlights the importance of the Stokes dissipation mechanism in the global oceans. It also highlights the need to incorporate this mechanism into existing wave models. Since the overall Stokes dissipation rate is more than the dissipation rate in the surf zones, it cannot be ignored and inclusion of the Stokes dissipation mechanism in wave models may very well improve the accuracy of swell prediction (Kantha 2006).

5. Concluding Remarks

We have provided a preliminary estimate of the Stokes dissipation rate of the surface gravity waves in the global oceans. The average for the year 2007 is about 2.53 TW, more than the 2.4 TW dissipation rate of wave energy in the surf zones around the ocean margins, and consequently too important to be ignored. This suggests that there is an urgent need to incorporate this mechanism into operational wave models so that the dissipation of wind waves, including the low frequency part of the wind wave spectrum (swell), can be more accurately represented and parameterized.

The energy extracted from waves is deposited in the mixed layer and unlike energy injected by breaking waves, which affects only the top few meters of the mixed layer, it affects deeper parts of the mixed layer leading to enhanced turbulence, more uniform velocity profiles and higher deepening rate of the mixed layer (Kantha and Clayson 2004, Carniel et al. 2005).

Needless to say that this study needs to be followed up by a more thorough in-depth study using a reliable mixed layer model (e.g. Kantha and Clayson 1994, 2004), the overall objective of this note being merely to point out the potential importance of the Stokes dissipation of surface gravity waves in the global ocean.

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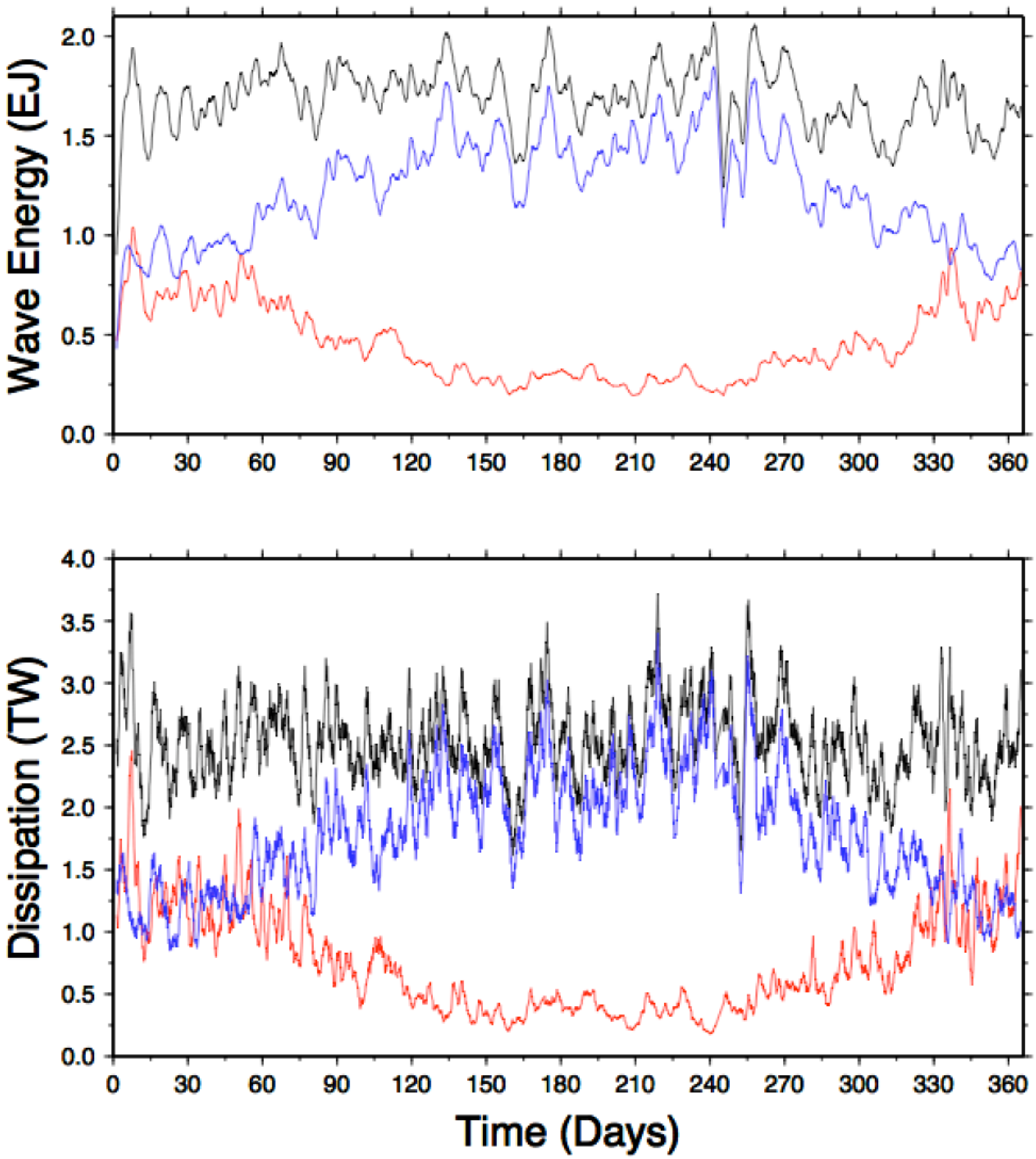

Wave Energy (EJ)
0.0
0.5
1.0
1.5
2.0
Dissipation (TW)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
30
60
90
120
150
180
210
240
270
300
330
360
Time (Days)

Figure 1a

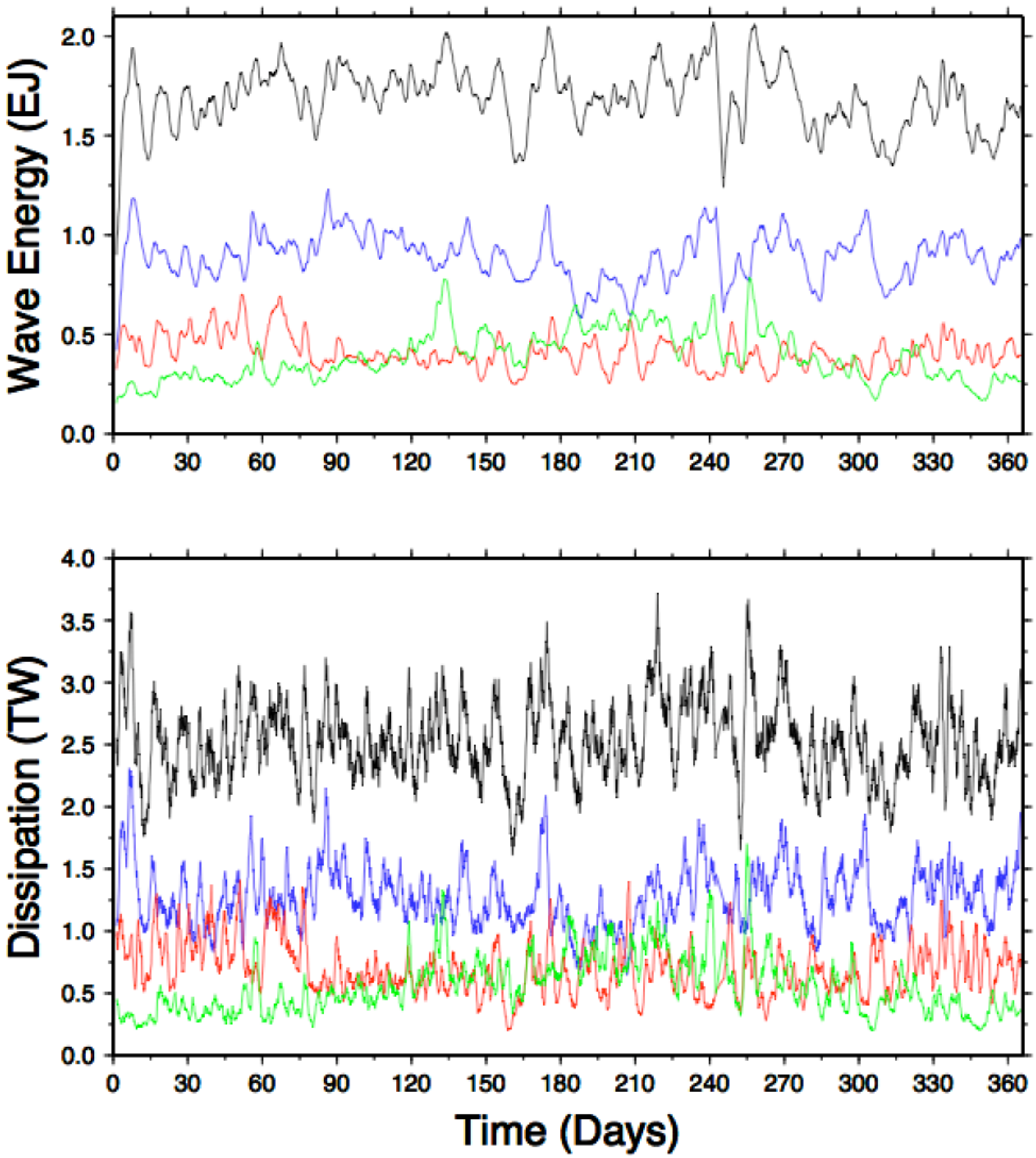
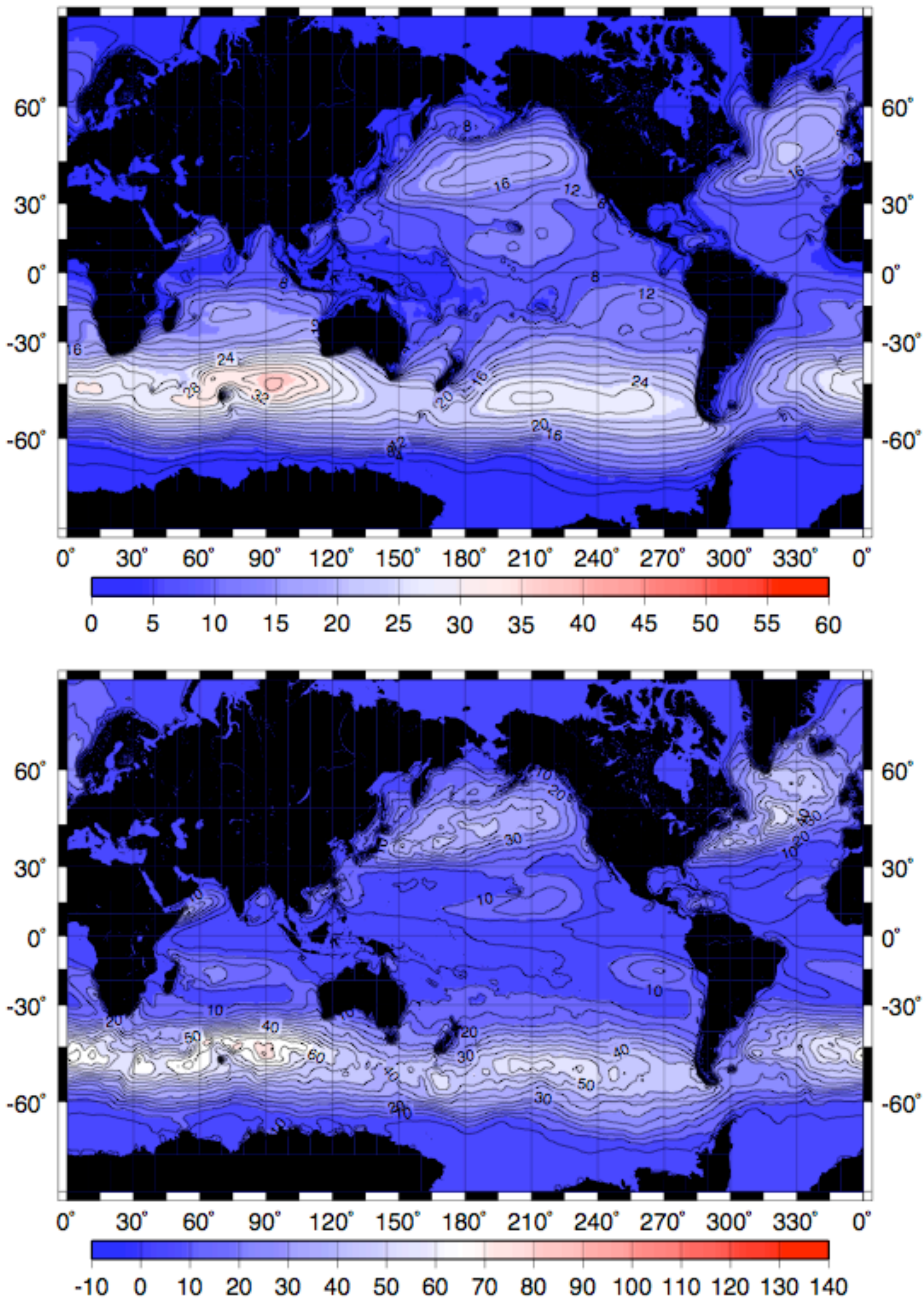

Wave Energy (EJ)
2.0
1.5
1.0
0.5
0.0
0
30
60
90
120
150
180
210
240
270
300
330
360
Dissipation (TW)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
30
60
90
120
150
180
210
240
270
300
330
360
Time (Days)

Figure 1b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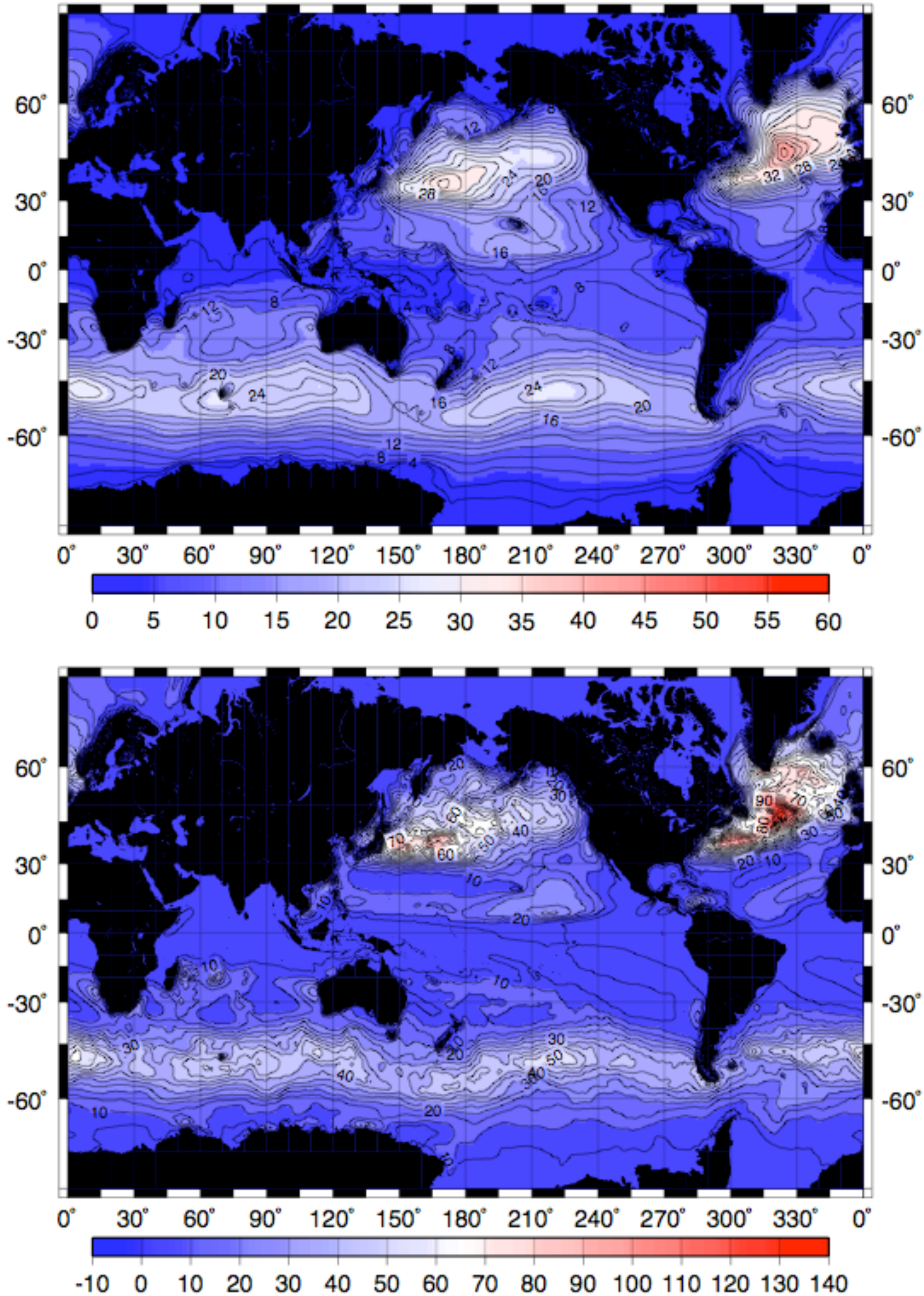
0° 30° 60° 90° 120° 150° 180° 210° 240° 270° 300° 330° 0°
0 5 10 15 20 25 30 35 40 45 50 55 60
60° 30° 0° -30° -60°
0° 30° 60° 90° 120° 150° 180° 210° 240° 270° 300° 330° 0°
-10 0 10 20 30 40 50 60 70 80 90 100 110 120 130 140

Figure 2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60° 30° 0° -30° -60°
0° 30° 60° 90° 120° 150° 180° 210° 240° 270° 300° 330° 0°
-10 0 10 20 30 40 50 60 70 80 90 100 110 120 130 140

Figure 3a

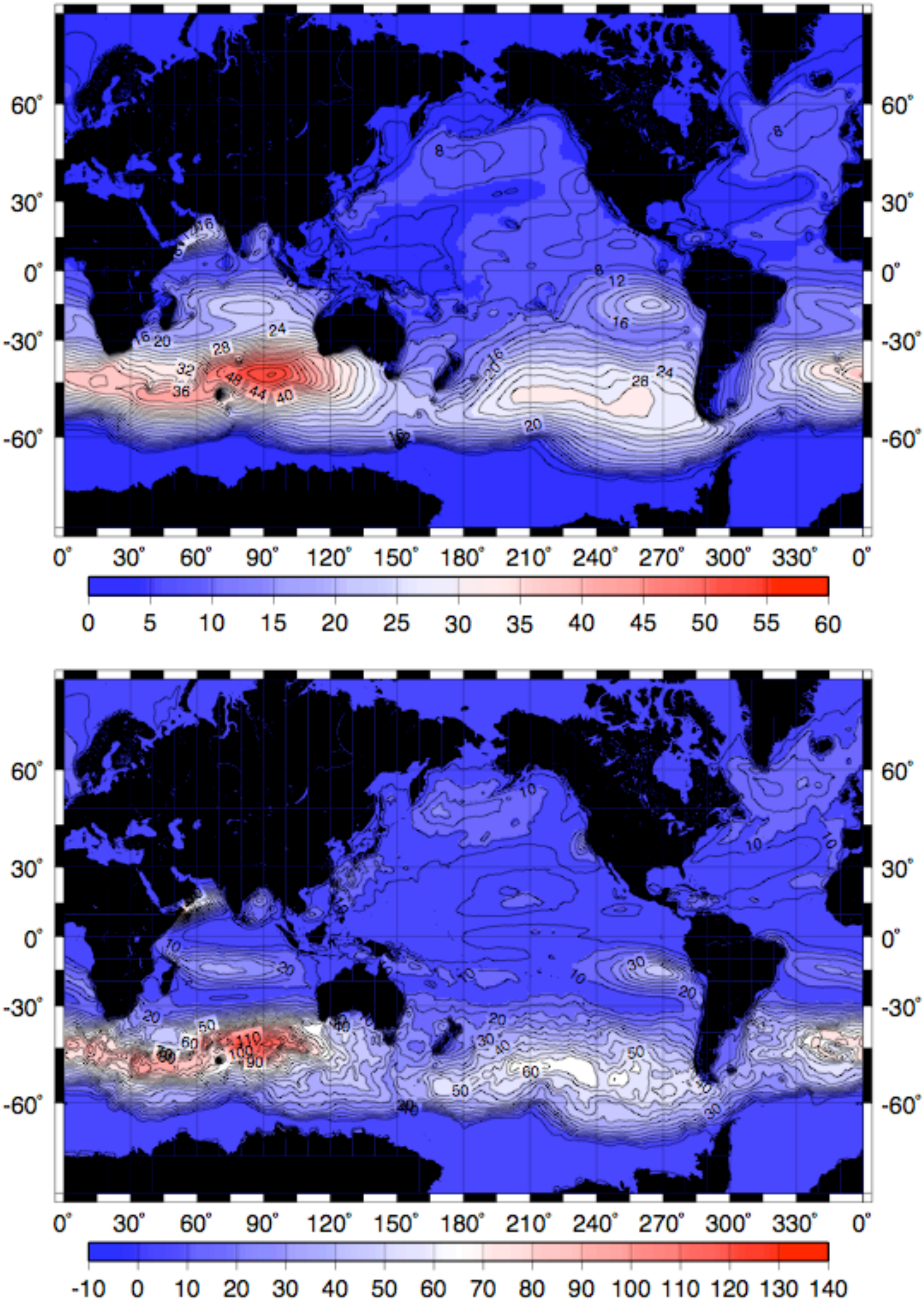
60°
30°
0°
-30°
-60°
0° 30° 60° 90° 120° 150° 180° 210° 240° 270° 300° 330° 0°
0 5 10 15 20 25 30 35 40 45 50 55 60
-10 0 10 20 30 40 50 60 70 80 90 100 110 120 130 140

Figure 3b

Region	Stokes dissipation rate (TW)	Wave energy (EJ)
	Minimum/Average/Maximum	Minimum/Average/Maximum
Global	1.61/2.53/3.72	0.90/1.68/2.07
North Hemisphere	0.18/0.74/2.46	0.20/0.46/1.04
South Hemisphere	0.85/1.79/3.40	0.43/1.23/1.85
Pacific sector	0.65/1.28/2.31	0.42/0.89/1.23
Atlantic sector	0.20/0.70/1.41	0.25/0.41/0.70
Indian sector	0.20/0.55/1.70	0.15/0.38/0.78

Table 1