Scientific report on the activity by Franco Bagnoli during the Short Term Mobility in Mexico

During my stay, With Dr. Rechtman we studied the problem of regional controllability of discrete systems (as declared in the proposal). In particular we studied the problem of controlling a region of a one-dimensional cellular automata by acting only on the periphery of that region. We also started writing a scientific article (to be submitted to Phys. Rev. E).

Cellular Automata (CA) are widely used for modelling physical problems, ranging from physics to biology, medicine, ecology and economics. The problem that we want to address here is that of forcing the appearance of a given pattern inside a region by imposing a suitable set of values to the sites that surround that region. We first defined the problem of a complete control of that region, i.e., being able to drive the systems (within the target region) towards a given state, for any initial configuration. We obtained some preliminary theorem about the possibility of imposing such a control. First, the boundary control problem can be mapped into an initial-value problem. Second, the target configuration should depend by a sufficient number of external sites in the initial configuration, and do not depend linearly from any initial site inside the region (otherwise it is impossible to impose a configuration for any initial one). Third, we can prove that for linearly peripheral CA (i.e., rules that depends linearly by peripheral sites), it is always possible to find a control, and we explicitly obtained this procedure.

For non-linear rule we were not able to obtain explicit conditions, so we evaluate numerically all preimages of any desired target configuration with an optimized code, so that we could evaluate the conditions for the control, but no particular pattern emerged.

In any case, the possibility of forcing the appearance of a given configuration does not imply the possibility of forcing the system to follow an arbitrary trajectory. Indeed, a simple count of available degrees of freedom shows that it is impossible to force a system to follow an arbitrary trajectory. So we devoted the second part of the study to the problem of inducing a system to synchronize with a “master” one, when both follow the same rule. In this case the trajectory of the “slave” system is a natural one, and the only problem is that of inducing the synchronization only acting on the boundary of the region.

We studied this problem using a continuously-tunable system, which is a system that depends on a quenched random field, and two tunable parameters. We previously studied this problem [F Bagnoli, N Boccara, R Rechtman, Nature of phase transitions in a probabilistic cellular automaton with two absorbing states, Physical Review E 63 (4), 046116], so that we already know its phase diagram and the location of the “chaotic” phase. We studied numerically the synchronization problem obtaining the relationship that links the value of parameters, the width of the region and the synchronization time. The results are consistent with the expectations, i.e., that the synchronization time depends exponentially on the width of the synchronizing region and that there is a critical divergence for the values of parameters for which the rule becomes linear, in which case the first method can be employed for synchronization.

Sesto Fiorentino, 29/11/2016

Franco Bagnoli