Quantum measurement and dynamics of open quantum systems

a report for the CNR short-term-mobility program

by

Paola Verrucchi

Istituto dei Sistemi Complessi ISC-CNR, Dipartimento di Fisica ed Astronomia, Università di Firenze, Italy

in collaboration with

Sabrina Maniscalco and Teiko Heinosaari

Department of Physics and Astronomy, University of Turku, Finland

(Dated: July 26, 2015)

In this report we briefly describe the analysis developed during the three weeks spent by P.V. at the Turku Centre for Quantum Physics (TCQP - Department of Physics and Astronomy of the Turku University, Finland) in the framework of the CNR-stm program mentioned in the title. Besides highlighting the obtained results, we also describe the research lines that have been identified as possibly leading to further advancements in our understanding of the quantum measurement process.

I. INTRODUCTION

An open quantum system (OQS) is a quantum system Γ (with Hilbert space $\mathcal{H}_\Gamma$), interacting with an equally quantum environment Ξ (with Hilbert space $\mathcal{H}_\Xi$). The total system $\Psi = \Gamma \cup \Xi$ has Hilbert space $\mathcal{H}_\Psi = \mathcal{H}_\Gamma \otimes \mathcal{H}_\Xi$.

The dynamics of OQS has recently emerged as a crucial topic in the study of quantum devices that require a controlled action on their components, due to the fact that such action is implemented by apparatuses playing the role of quantum, albeit "big", environments. The specific attention consequently payed to the quantum evolution of a system interacting with its macroscopic environment has made clear that it is indeed in the peculiarities of one such evolution that the most debated issues in the theory of quantum mechanics might find their solution. This is the case, in particular, of what is usually dubbed as *emergence of classicality*, i.e. the mechanism that makes us to observe a classical reality, despite the fundamental laws of physics being quantum mechanical.

The idea that the emergence of classicality be related with the peculiar features of the quantum measurement process already appears in the very first discussions on the foundations of quantum mechanics. However, taking apart the neverending diatribe between different interpretations of such theory, the idea has been given a formal content only some decades ago, after the definition of proper mathematical to ols for the analysis of the OQS dynamics. In fact, it is nowadays recognized that a quantum measurement is a dynamical process involving a principal system Γ (the observed object) and its environment Ξ (the measuring apparatus), and that the original scheme w proposed by von Neumann[1–3] for describing quite a specific type of measurement in terms of the unitary propagator relative to the isolated system $\Psi = \Gamma \cup \Xi$, can indeed be replaced by a more general treatment, where completely-positive trace-preserving (CPTP) dynamical maps are used for analysing the evolution of Γ[2, 4]. The latest developments in the field have made it clearer, though not unanimously accepted, that we experience a classical reality due to the continuous interaction between each microscopic component of any physical system and its environment, to be considered macroscopic as far as it contains ourselves as observers. The above argument, and its several formal expressions, still leaves two fundamental questions open:

i) why should the infinitely many different types of interaction between Γ and Ξ give always rise to a measure-like dynamics, and

ii) why different observers, even if extracting information from different sections of Ξ, actually see the same world.

In order to better grasp the meaning of the above questions and analyse a phenomenon recently proposed[5] for providing them with a formal answer, namely the "quantum darwinism", in Sec. II we describe what is usually meant by measure-like dynamics, in Sec. III we show how quantum darwinism can emerge as a generic phenomenon, both from an information-theoretic and a more physical viewpoint, and in Sec. IV we propose a way of understanding such generality in the framework of a parametric description of the quantum measurement process.

II. MEASURE-LIKE DYNAMICS

Let us consider the measurement process originally introduced by von Neumann[1] and later characterized by Ozawa[6] under the name of *conventional measuring process* of non-degenerate sharp observables. Its initial step corresponds to a unitary evolution of Ψ that is determined, in the so called *standard model* [2], by the unitary propagator $V_t \equiv e^{-it\hat{H}_\Psi}$ with

$$\hat{H}_\Psi = g\,\hat{O}_\Gamma \otimes \hat{O}_\Xi + \hat{\mathbb{I}}_\Gamma \otimes \hat{H}_\Xi\ , \qquad (1)$$

where g is the coupling constant, $\hat{O}_\Gamma$ is the hermitian operator on $\mathcal{H}_\Gamma$ associated to the measured observable, while $\hat{O}_\Xi$ is the operator on $\mathcal{H}_\Xi$, conjugate to the one that is usually referred to as *the pointer* ; moreover, $\hat{H}_\Xi$ acts on Ξ only, $\hat{\mathbb{I}}_\Gamma$ is the identity operator on $\mathcal{H}_\Gamma$, and we have set $\hbar$=1.

An essential assumption in the conventional measuring process is that the initial state of Ψ be separable, $|\Psi(0)\rangle = |\Gamma\rangle \otimes |\Xi\rangle$, so that the result of the measure can be ascribed to the system Γ in the pure state $|\Gamma\rangle$. Writing $\Gamma = \sum_\gamma c_\gamma|\gamma\rangle$ with $\{|\gamma\rangle\}_\Gamma$ the $\mathcal{H}_\Gamma$-orthonormal basis of the $\hat{O}_\Gamma$ eigenstates, $\hat{O}_\Gamma|\gamma\rangle = \omega_\gamma|\gamma\rangle$, from Eq.(1) it follows

$$|\Psi(t)\rangle = V_t|\Psi(0)\rangle = \sum_\gamma c_\gamma|\gamma\rangle \otimes |\Xi^\gamma(t)\rangle \tag{2}$$

with

$$|\Xi^\gamma(t)\rangle \equiv e^{-it\hat{H}^\gamma_\Xi}|\Xi\rangle \ , \tag{3}$$

and

$$\hat{H}^\gamma_\Xi \equiv g\,\omega_\gamma\hat{O}_\Xi + \hat{H}_\Xi \ . \tag{4}$$

Given Eq.(2) the reduced density matrix for the apparatus reads

$$\rho_\Xi(t) = \sum_\gamma |c_\gamma|^2 \ |\Xi^\gamma(t)\rangle\langle\Xi^\gamma(t)| \ . \tag{5}$$

What characterizes Eq.(2) as a measure-like evolution of Ψ, besides the initial separable state, is the existence of the basis $\{|\gamma\rangle\}_\Gamma$ such that Eqs.(2)-(4) hold. This condition, trivially ensured in the standard model that contains only one operator $\hat{O}_\Gamma$, does not generally characterize the interaction between Γ and Ξ: in fact, one such interaction might well involve several non-commuting operators on Γ, implying the non-existence of a basis of common eigenvectors, and therefore the impossibility of defining $\hat{H}^\gamma_\Xi$ and obtain $|\Xi^\gamma(t)\rangle$ as in Eq.(3).

On the other hand, it is just the appearance of the states $|\Xi^\gamma(t)\rangle$, whose γ-dependence establishes a one-to-one correspondence with the states $|\gamma\rangle$ and the coefficients c_γ, that allows one to recognize, in Eq.(2), the evolution capable of returning an outcome ω_γ, with probability $|c_\gamma|^2$; in fact, this is obtained via the Ξ-positioning into the state $|\Xi^\gamma(t)\rangle$, for any time $t > \tau_\mathrm{d}$, where τ_d is the time after which decoherence is observed in Γ, with respect to the basis $\{|\gamma\rangle\}_\Gamma$ (see Ref. [7] for a more detailed discussion of the role of decoherence in the measurement process).

III. QUANTUM DARWINISM IS GENERIC

A. an information-theoretic viewpoint

Let us now concentrate on the questions *i)-ii)* raised in Sec. I. Referring to a recent paper by Brandao, Piani and Horodecki [8], we rephrase their **Theorem 1.** as follows:

- consider a system A with an environment B, on its own made by a number of subsystems B_i, i.e.
$B = \cup_i^n B_i$;
- let the initial state of $A \cup B$, $|AB(0)\rangle$, evolve according to an utterly generic physical dynamics $|AB(t)\rangle = U_t|AB(0)\rangle$, with U_t unitary $\forall t$;
- choose a number $0 < \delta < 1$:

- it always exists a system $B_S \equiv \cup_{i\in S}B_i$ with $|S| \geq n(1-\delta)$, i.e. a subsystem of B made of a selection of at least $(n-n\delta)$ parts of it, such that the dynamics of each $B_j \in B_S$ tends to that corresponding to a measure-like evolution when n goes to infinity. The measured quantum observable regards the system A, and it is the same for all the B_j in B_S.

This is formally expressed by the relation

$$||\Lambda_j - \mathcal{E}_j||_\diamond \leq 3\mathrm{d}_A^2\left(\frac{\ln(2)\log(\mathrm{d}_A)}{n\delta^3}\right)^{\frac{1}{3}} \tag{6}$$

where $||\ ||_\diamond$ is a norm (the *diamond*-norm) that can be used for quantifying the difference between CPTP dynamical maps, d_A is the dimension of $\mathcal{H}_A$, while the CPTP maps Λ_j and $\mathcal{E}_j$ define two different reduced dynamics for B_j, according to

$$\Lambda_j[\rho_{B_j}(0)]_t = \mathrm{Tr}_{A\cup B\setminus B_j}\left[U_t|AB(0)\rangle\langle AB(t)|U_t^\dagger\right] \tag{7}$$

and

$$\mathcal{E}_j[\rho_{B_j}(0)]_t = \sum_\gamma |c_\gamma|^2|B_j^\gamma(t)\rangle\langle B_j^\gamma(t)| \ , \tag{8}$$

where $\rho_{B_j}(0) = \mathrm{Tr}_{A\cup B\setminus B_j}|AB(0)\rangle\langle AB(0)|$. Notice that while Λ_j follows from any generic global evolution, $\mathcal{E}_j$ specifically corresponds to a measure-like one, as seen by comparing Eq.(8) with Eq. (5).

The above result is obtained, discussed and interpreted in Ref. [8] by an information-theoretic approach that, while ensuring the beauty of a most general picture, does not immediately translate into the physical language needed for understanding its possible consequences on the actual functioning of quantum devices. To this purpose we aim at formally relating **Theorem 1.** of Ref. [8] with the concept of quantum darwinism as introduced by Zurek in Ref. [5], and the description of quantum measurements recently proposed in Refs. [9, 10]. In the next sub-section we show how, after setting the problem as suggested by the approach sketched in Sec. II, a more physical understanding naturally emerges.

B. a more physical viewpoint

The first difference that one notices when comparing Sec. II and Sec.IIIa is that, despite both concern a composite bipartite system, $\Gamma \cup \Xi$ and $A \cup B$ respectively, only in the former case the notion of observed system

(Γ) and measuring apparatus (Ξ) is explicit. This is not a moot point, as it goes together with the fact that the initial state of Γ and Ξ must be taken pure, while no such hypothesis is made on A and B.

This observation gives us a clue on the role played by the subsystems A, B, and $\Delta \equiv B\backslash B_S$ of Sec. IIIa as compared with that of Γ and Ξ in Sec. II: in fact, setting

$$\begin{aligned} \Psi &= \Gamma \cup \Xi = A \cup B \ , & (9)\\ \Gamma &= A \cup \Delta \ , & (10)\\ \Xi &= B\backslash\Delta = B_S \ , & (11) \end{aligned}$$

we understand Δ as the system that purifies the initial state of A, thus guaranteeing that the observed system $\Gamma = A \cup \Delta$ is in the pure state $|\Gamma\rangle$ at t=0. In other terms: A cannot be considered as an observed system, at least because it is not initially prepared in a pure state. However, there might exist a subsystem Δ in B, such that the state of $A \cup \Delta$ at $t = 0$ is pure, and can hence be considered as the initial state of the observed system in a measurement process. Notice that the parameter δ that defines the maximum number $n\delta$ of subsystems B_i forming B_S, i.e. those that do not obey the inequality (6), turns out to be related with the dimension of the Hilbert space of the ancillary system Δ. Therefore, we interpret the evidence that a small δ requires a large n for the inequality (6) to be meaningful, as follows: any dynamics of an OQS is doomed to be that of an observed system if the Hilbert space of its environment is very large, ensuring $n\delta^3 \gg 1$ for any finite $\delta < 1$.

IV. ANY PHYSICAL EVOLUTION OF AN OQS WITH A MACROSCOPIC ENVIRONMENT IS A QUANTUM MEASUREMENT PROCESS

Inspired by the above reasoning, while closely retracing the derivation of **Theorem 1.** in Ref.[8], we propose a possible demonstration of this Section's title according to the following guidelines.

Consider the system Ψ, made of $\Gamma = A \cup \Delta$ and $\Xi = B\backslash\Delta$, and assume it is initially prepared in the separable state $|\Gamma\rangle \otimes |\Xi\rangle$. As discussed in the previous section, and given that Ψ is isolated, this assumption can always be made by properly choosing Δ. Notice, however, that B and Δ must be substantially different for Γ and Ξ to keep their role of principal system and environment.

Consider the Schmidt decomposition of $|\Psi(t)\rangle$ at some time t

$$|\Psi(t)\rangle = \sum_\gamma c_\gamma |\gamma\rangle \otimes |\Xi^\gamma\rangle \ , \quad (12)$$

where we have dropped the time-dependence of $c_\gamma, |\gamma\rangle$, and Ξ^γ, for the sake of a lighter notation: We define the two related states

$$\begin{aligned} |\Psi(t)\rangle_{\max} &= \sum_\gamma c_\gamma |\gamma\rangle \otimes |\Xi^\gamma\rangle_{\max} \ , & (13)\\ |\Psi(t)\rangle_{\rm sep} &= \sum_\gamma c_\gamma |\gamma\rangle \otimes |\Xi^\gamma\rangle_{\rm sep} \ , & (14) \end{aligned}$$

where

$$\begin{aligned} |\Xi^\gamma\rangle_{\max} &= \frac{1}{\mathrm{m}} \sum_{k=1}^{m} \otimes_{l=1}^{\mathrm{d}_\Xi} |\xi_k^\gamma\rangle_l \ , & (15)\\ |\Xi^\gamma\rangle_{\rm sep} &= \otimes_{l=1}^{\mathrm{d}_\Xi} |\xi^\gamma\rangle_l \ , & (16) \end{aligned}$$

with $m \leq \mathrm{d}_\Xi$ and $_l\langle\xi_k^\gamma|\xi_{k'}^\gamma\rangle_l = \delta_{kk'}$. The corresponsing density matrices for any subsystem B_j of Ξ are

$$\begin{aligned} \rho^j &= \sum_\gamma |c_\gamma|^2 \left[\mathrm{Tr}_{\Xi\backslash B_j} |\Xi^\gamma\rangle\langle\Xi^\gamma|\right] \ , & (17)\\ \rho^j_{\max} &= \sum_\gamma |c_\gamma|^2 \frac{1}{\sqrt{m}} \sum_{k=1}^{m} |\xi_k^\gamma\rangle_j \ {}_j\langle\xi_k^\gamma| \ , & (18)\\ \rho^j_{\rm sep} &= \sum_\gamma |c_\gamma|^2 |\xi^\gamma\rangle_j \ {}_j\langle\xi^\gamma| \ , \ . & (19) \end{aligned}$$

In the state (19) we recognize the structure (5) expected for a measure-like dynamics, where B_j is the apparatus, and the operator for the measured observable is any hermitian $\hat{O}_\Gamma$ with eigenvectors the elements of the Schmidt basis, $\{|\gamma\rangle\}_\Gamma$ in Eq.(12), at time t . Therefore, we could accomplish our goal were we able to demonstrate that ρ^j gets the structure of $\rho^j_{\rm sep}$ as Ξ becomes macroscopic. Notice that, as neither the basis $\{|\gamma\rangle\}_\Gamma$ nor the coefficients c_γ depend on j, the above demonstration would provide an answer not only to question *i)* but also to question *ii)*, of Sec. I. On the other hand, the time-dependence of the Schmidt basis used in Eq.(12) is somehow puzzling, as it suggests that the observable being measured is different at different times, which is not a known feature of quantum darwinism, so far.

V. FURTHER DEVELOPMENTS

Without entering into the formal structure of the demonstration mentioned in the previous section, we end this report by briefly describing its guidelines and main ingredients. We first focus upon the above mentioned difference between ρ^j and $\rho^j_{\rm sep}$, and understand that it is now necessary to choose one, amongst the several, definition of distance $d(\rho, \rho')$ between any two quantum states. We then consider that our interest is in evaluating $d(\rho^j, \rho^j_{\rm sep})$ when the the dimension of $\mathcal{H}_\Xi$, with $\Xi = \cup_{i\in S} B_i$, goes to infinity. This suggests us to make use of a recently proposed[11] parametric representation with environmental coherent states, which is specifically designed for studying OQS whose environment needs being considered through its quantum-to-classical crossover. By this formalism, the reduced density matrix of the environment is studied in terms of its Husimi function, which is a positive and normalized distribution $\chi^2(\Omega)$ on the differentiable manifold $\mathcal{M}$, whose definition follows from the construction of the environmental coherent states $|\Omega\rangle$. The distance between states can thus be analysed by referring to the Monge distance[12] between distributions, as done for studying the decoherence process in Refs [9, 13]. As a further essential ingredient of our treatment we recognize the concept of "classical equivalence", as introduced by Yaffe in Ref. [14] and specifically used in describing the quantum-measurement process in Ref. [10], and its relation with the existence of a global environmental symmetry, such as that enforced in Ref.[15] for studying the specific model there considered.

The reasoning behind the demonstration we are working on is the following: the Monge distance between $\chi^2(\Omega_j)$ and $\chi^2_{\rm max}(\Omega_j)$ is shown to vanish as Ξ becomes macroscopic, and, in the very same limit, the states represented by $\chi^2_{\rm max}(\Omega_j)$ and $\chi^2_{\rm sep}(\Omega_j)$ become classically equivalent, due to their deriving from global environmental states featuring the same symmetry. Therefore, the generic distribution $\chi^2(\Omega_j)$ has the same classical limit of the one, $\chi^2_{\rm sep}(\Omega_j)$, corresponding to a measure-like dynamics of Ψ.

As a final comment we add that the result we aim at demonstrating in general, as described above, has been recently confirmed in the specific model treated in Ref. [16], thus reinforcing our confidence in the possibility of accomplishing our goal.

VI. ADDITIONAL INFORMATION ON THIS CNR-STM PROGRAM

The material presented in this report resulted from the joined consideration of the theory of quantum operations, quantum measurement process, and OQS dynamics, which has been made possible by a systematic sharing of the personal expertise of Dr. Teiko Heinosaari, Dr. Paola Verrucchi, and Prof. Sabrina Maniscalco, respectively. On the other hand, the ideas expressed in Sec.III took their present form thanks to discussions, group-meetings, and department seminars, involving all the members of the TCQP, that have positively characterized the three weeks of this CNR-stm program. Although the work here presented needs being finalized, particularly as far as the demonstration mentioned in Sec.IV is concerned, important aspects of the quantum measurement process as an OQS evolution have already been understood, and we plan to let this collaboration continue well beyond the completion of this specific work.

[1] J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, 1996).
[2] P. Busch, J. P. Lathi, and P. Mittelstaedt, *The quantum theory of measurement* (Springer-Verlag, Berlin, 1996).
[3] M. Schlosshauer, *Decoherence and the Quantum-To-Classical Transition*, The Frontiers Collection (Springer, 2007).
[4] T. Heinosaari and M. Ziman, *The mathematical language of Quantum Theory* (Cambridge University Press, 2012).
[5] W. H. Zurek, Nature Physics **5**, 151 (2009).
[6] M. Ozawa, J. Math. Phys. **25**, 292 (1984).
[7] P. Liuzzo-Scorpo, A. Cuccoli, and P. Verrucchi, Int. J. Theor. Phys. (2015).
[8] M. P. Fernando G. S. L. Brandao and P. Horodecki, arXiv: p. 1310.8640 (2013).
[9] P. Liuzzo-Scorpo, Master thesis, Università degli studi di Firenze (2014).
[10] P. Liuzzo-Scorpo, A. Cuccoli, and P. Verrucchi, European Physical Letters, in print (2015).
[11] D. Calvani, A. Cuccoli, N. I. Gidopoulos, and P. Verrucchi, Proceedings of the National Academy of Sciences **110**, 6748 (2013).
[12] K. Zyczkowski and W. Slomczynski, Journal of Physics A: Mathematical and General **31**, 9095 (1998).
[13] P. Liuzzo-Scorpo, A. Cuccoli, and P. Verrucchi, In preparation (2015).
[14] L. G. Yaffe, Rev. Mod. Phys. **54**, 407 (1982).
[15] G. Chiribella and G. M. D'Ariano, Phys. Rev. Lett. **97**, 250503 (2006).
[16] C. Foti, Master thesis, Università degli studi di Firenze (2015).