Rapid transit network design with modal competition

Abstract

We present a mixed-integer linear program for the rapid transit network design problem with static modal competition. Previous mathematical programs eluded modal competition because of the excessive complexity of modeling cost differences for each flow in the network. We overcome this difficulty by exploiting a pre-assigned topological configuration.

Keywords: Rapid transit network design; modal competition

1 Introduction

We extend the rapid transit network design problem (RTNDP) of Gutiérrez-Jarpa et al. (2013) by introducing modal competition. In Gutiérrez-Jarpa et al. (2013) an origin-destination flow is considered as captured by the metro if stations are sufficiently close to the origin and the destination of the flow. We observe that by maximizing the captured traffic according to this criterion results in improving *access*, i.e. the number of citizens that *could* enjoy the metro network for their daily trips. Access is a worthwhile objective in public transit planning, but alone inadequate in reflecting modal choices. In this paper we consider a traffic flow as captured if the generalized cost of a metro trip is less than the cost by a car trip. This feature has been eluded in previous discrete mathematical programs because of the complexity of considering origin-destination

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flows. As observed by Marín and García-Ródenas (2009), this feature would require adding a multi-commodity formulation, where each flow is considered as a distinct commodity. We overcome this difficulty by exploiting a pre-assigned topological configuration. As motivated by Bruno and Laporte (2002); Bruno et al. (2002), a pre-assigned topological configuration is itself a positive feature for planners, hence this approach should be valuable in practice. Section 2 presents the mixed-integer linear program.

2 Mixed-integer linear program

We define the RTNDP on an undirected graph $G(N, E)$, where $N = \{1, ..., n\}$ is a node set and $E = \{(i, j) : i, j \in N, i < j\}$ is an edge set. A node represents a centroid of demand, and some nodes are also candidate stations. Each node i is characterized by a neighborhood $N(i) \subset N$ of sufficiently close candidate stations. An edge is a candidate rail link between stations. Edges are defined such that minimal and maximal distances between stations are enforced. The network is embedded in a pre-assigned topological configuration. We refer to Gutiérrez-Jarpa et al. (2013) for examples of basic configurations such as the star, the triangle, and the cartwheel. The configuration defines the number of transfer points so that the transit network is connected and passenger can transfer from one line to another. Each line is made up of segments which are chains of edges connecting two transfer points, or a transfer point with a end-point, a terminus. The output of the model is a set of segments. The combination of segments into lines is beyond the scope of this study since it pertains to tactical evaluations of train scheduling and offered capacity. Let S be the set of corridors where segments are embedded. We define by $N_s \subseteq N$ the set of nodes of corridor $s \in S$, and $N^S = \cup_{s \in S} N_s$. The extreme nodes of a segment must belong to a predefined extreme set, the set of nodes that are candidates to the location of a terminus or a transfer station. Let T_k denote the k-th extreme set, and O_k be the set of all corridors having one of their extreme nodes in T_k. We indicate by a and b the indices of an origin and a destination, respectively. We observe that given a topological configuration we can approximatively determine the set of candidate edges for a minimum cost path between a and b in the rapid transit network. Let $P_{ab} \subset E$ be this approximate set. Thanks to this approximation we can define the generalized cost of a path in the rapid transit network without an explicit multi-commodity formulation. For notational simplicity and model compactness we assume symmetry, i.e. the travel cost from a to b is equal to that from b to a. This assumption can be easily removed and is not a limitation of the proposed model. In the following we define the data and the variables required by the mathematical program.

An edge is characterized by the following attributes:

- d_{ij}, distance between i and j.
- c_{ij}, construction cost of the infrastructure between station i and station j. It includes the cost of building a station as follows: if i or j is a candidate terminus the full cost of the station is part of c_{ij}; if i or j is a candidate transfer station a quota of the building cost is included in c_{ij} according to the number of pertaining segments; otherwise, half of the station cost is included in c_{ij}.
- τ_{ij}, passenger travel cost from station i to station j. It includes the cost of travel time between i and j, and the time lost due to stopping at j.

A pair of nodes, not necessarily an edge, is characterized by the following attributes:

- t_{ab}, traffic flow between a and b.
- T_{ab}, generalized cost for a passenger traveling between a to b by car.
- δ_{ak}, passenger access or egress cost between the node a and the station k.
- ϕ_{ab}, fixed cost for a passenger traveling between a and b by the rapid transit network. It includes the cost of time lost for transfer, if any.
- M^+_{ab}, difference between T_{ab}, trip cost by car, and the minimum cost path between a and b in the residual network defined by the approximate edge set P_{ab}, and considering access, egress, and fixed costs as defined above. By definition this value is positive, otherwise the pair (a, b) would not be considered in the RTNDP. M^+_{ab} represents an upper bound on the cost advantage of using the metro instead than the car.
- M^-_{ab}, difference between the maximum cost path between a and b in the residual network defined as above, and T_{ab}.

We introduce the following variables:

- x^s_{ij}, equal to 1 if the edge (i, j) of the corridor s is active, zero otherwise.

- β_{ab}, positive if the (a, b) flow is captured, zero otherwise; in the former case β_{ab} is the difference between the generalized car cost and the generalized transit cost for a passenger.

- y_i, equal to 1 if the station i belonging to an extreme set is activated, zero otherwise.

- w_i^s, equal to 1 if the station i belonging to a corridor s is activated, zero otherwise.

- z_{ik}, equal to 1 if the node i is served by the station k, zero otherwise.

- v_{ij}, equal to 1 if there is a path in the rapid transit network between i and j, zero otherwise.;

- r_{ab}, equal to 1 if the (a, b) flow is captured, zero otherwise.

For notational simplicity we introduce the set $\Psi \subset N \times N$ which contains the forbiden pairs of stations according to the minimal distance criterion, i.e. (i, j) belongs to Ψ if $d_{ij} \leq \hat{d}$, where $\hat{d}$ is the minimal distance between stations.

The mixed-integer linear program follows.

$$\text{minimize } Z_1 = \sum_{s \in S} \sum_{i,j \in N_s : (i,j) \in E} c_{ij} x_{ij}^s \tag{1}$$

$$\text{minimize } Z_2 = \sum_{a,b \in N : a < b} t_{ab} \beta_{ab} \tag{2}$$

subject to

$$\sum_{i\in T_k}\sum_{j\in N_s\setminus T_k}(x^s_{ij}+x^s_{ji})\geq 1 \qquad \forall k\in K, s\in O_k, \tag{3}$$

$$\sum_{j\in N_s}(x^s_{ij}+x^s_{ji}) = 2w^s_i - y_i \qquad \forall s\in S, i\in N_s, \tag{4}$$

$$\sum_{i\in T_k} y_i = 1 \qquad \forall k\in K, \tag{5}$$

$$w^s_i + w^s_j \leq 1 \qquad \forall s\in S, \text{ and } (i,j)\in\Psi, \tag{6}$$

$$\sum_{k\in N(i)} z_{ik}\leq 1 \qquad \forall i\in N, \tag{7}$$

$$z_{ik}\leq \sum_{s\in S} w^s_k \qquad \forall i\in N, \tag{8}$$

$$v_{ij}\leq \sum_{k\in N(i)} z_{ik} \qquad \forall i,j\in N: i<j, \tag{9}$$

$$v_{ij}\leq \sum_{k\in N(j)} z_{jk} \qquad \forall i,j\in N: i<j, \tag{10}$$

$$\begin{aligned}T_{ab} - [\sum_{s\in S_{ab}}\sum_{(i,j)\in P_{ab}}\tau_{ij}x^s_{ij} + \sum_{k\in N(a)}\delta_{ak}z_{ak} + \\ + \sum_{k\in N(b)}\delta_{bk}z_{bk} + \phi_{ab}] \leq M^+_{ab}r_{ab} \qquad \forall a,b\in N: a<b,\end{aligned} \tag{11}$$

$$\begin{aligned}\beta_{ab}\leq T_{ab} - [\sum_{s\in S_{ab}}\sum_{(i,j)\in P_{ab}}\tau_{ij}x^s_{ij} + \sum_{k\in N(a)}\delta_{ak}z_{ak} + \\ + \sum_{k\in N(b)}\delta_{bk}z_{bk} + \phi_{ab}] + M^-_{ab}(1-r_{ab}) \qquad \forall a,b\in N: a<b,\end{aligned} \tag{12}$$

$$\beta_{ab}\leq M^+_{ab}r_{ab} \qquad \forall a,b\in N: a<b, \tag{13}$$

$$r_{ab}\leq v_{ab} \qquad \forall a,b\in N: a<b, \tag{14}$$

$$\sum_{i,j\in Q} x^s_{ij}\leq \sum_{t\in Q\setminus\{q\}} w^s_t \qquad \forall s\in S, Q\subseteq N_s, q\in Q: |Q|>2, \tag{15}$$

$$x^s_{ij}\in\{0,\,1\} \qquad \forall (i,j)\in E, s\in S, \tag{16}$$

$$v_{ij}\in\{0,\,1\} \qquad \forall i,j\in N: i<j, \tag{17}$$

$$y_i\in\{0,\,1\} \qquad \forall k\in K, i\in T_k, \tag{18}$$

$$w^s_i\in\{0,\,1\} \qquad \forall s\in S, i\in N_s, \tag{19}$$

$$z_{ik}\in\{0,\,1\} \qquad \forall i\in N, k\in N(i), \tag{20}$$

$$r_{ab}\in\{0,\,1\} \qquad \forall a,b\in N: a<b, \tag{21}$$

$$\beta_{ab}\geq 0 \qquad \forall a,b\in N: a<b. \tag{22}$$

Objective (1) minimizes construction cost, and objective (2) maximizes the value of the captured flows. Constraints (3) require each segment with an extreme in site k to have an edge connecting a node in the site with a node in the corridor. Constraints (4) define continuity for each segment, i.e. if a node is the extreme of a segment then it must have one incident edge since $y_i = 1$ when i belongs to the extreme sets; otherwise it must have two incident edges ($y_i = 0$). Constraints (5) state that exactly one node belongs to an extreme set. Constraints (6) enforce minimal distances between non adjacent stations. Constraints (7) state that a node can be assigned to at most one station belonging to its neighborhood, and constraints (7) enforce that such assignment must activate the binary variable z_{ik}. Constraints (9) and (10) prevent traffic between two nodes to be considered unless there is active path in the rapid transit network. Constraints (11) , (12), and (13) define the variables β_{ab} and r_{ab} as explained in the following. Whenever the cost difference between the car and the rapid transit is positive for the flow (a, b) the constraints (11) force r_{ab} to be equal to one. Henceforth, β_{ab} is equal to the mentioned cost difference by constraints (12) and the maximization of objective (2). Whenever the cost difference between the two modes is negative, i.e. the car is more advantageous, then the flow is not captured and the non-negativity of variable β_{ab} and constraints (12) force r_{ab} to be equal to zero. The value of β_{ab} is then forced to zero by (13). Constraints (14) state that a flow cannot be captured unless there is an active path for it. Constraints (15) eliminate subtours from the solution. The remaining constraints define the range of the variables.

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