Relazione Scientifica

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1.1 Titolo del programma di ricerca:

"Analisi statistica con metodologie avanzate della dinamica temporale in serie storiche di tremore sismico associato ad attività vulcanica"

1.2 Keywords : El-Hierro, metodi statistici, tremore sismico

1.3 Dipartimento: Terra e Ambiente

1.3 Durata: 21 giorni

1.4 Periodo: dal 28/11/2013 al 18/12/2013

2. PERFORMED ACTIVITIES

During the research stay at Instituto de Ciencias de la Tierra Jaume Almera (CSIC), Barcelona, the research activity was devoted to the statistical analysis of the time series of the continuous seismic signal recorded from 15/07/2011 to 29/02/2012 at El-Hierro (Canary Islands) (Fig. 1). The seismic tremor ground motion amplitude (RSAM) (counts) was computed in the vertical seismic component by averaging the modulus of the signal. Continuous seismic data were subdivided into windows of 600 s (10 min at 50 Hz) with no overlap; in each window the integral of the modulus of the signal was computed after removing the instrumental offset (Fig. 2). Within the observation period the volcano was characterized by an eruptive phenomenon started on 10 October 2011 that lasted few weeks.

The statistical analyses performed on these data were:

1) multifractal, by using the Multifractal Detrended Fluctuation Analysis;

2) entropic-informational, by using the Fisher-Shannon method.

The details of the performed analysis along with the obtained results are described in the following sections 2.1 and 2.2.

2.1 Multifractal Analysis

2.1.1 The method

The multifractal analysis was performed by means of the Multifractal Detrended Fluctuation Analysis (MF-DFA) (Kantelhardt et al., 2002), which is an effective tool to characterize multifractality in nonstationary data. The method is based on the well-known detrended fluctuation analysis (Peng et al., 1995). Considered the time series $x(i)$, with $i=1,2,\ldots,N$ and N the length of the series, x_{ave} indicates the mean of the series. Assuming that x(i) are increments of a random walk process around the average x_{ave}, the "trajectory" or "profile" is obtained by integrating the signal

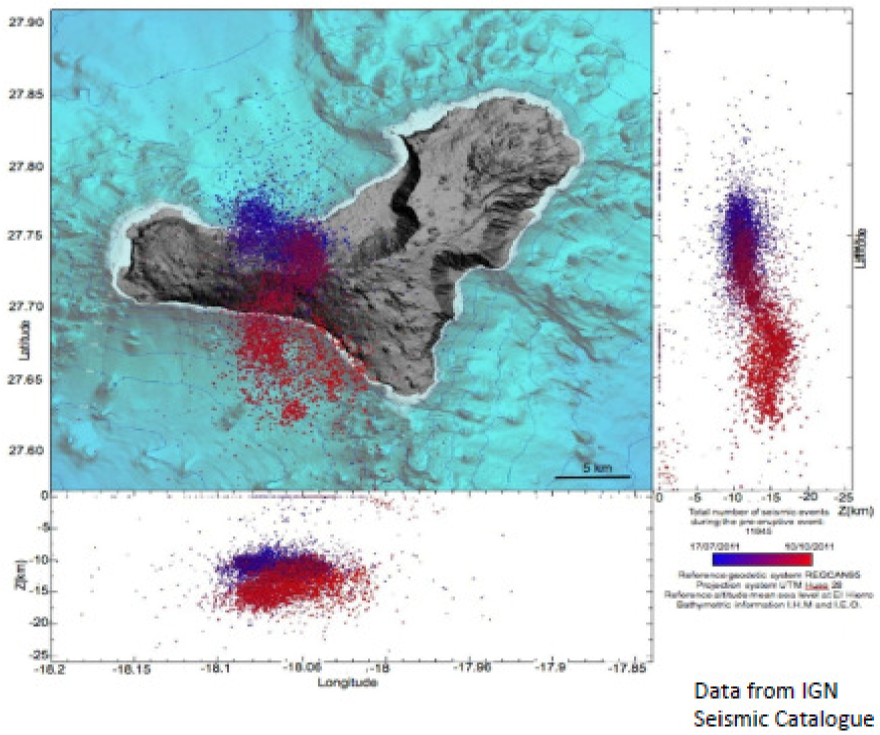

Fig. 1. Map of the El-Hierro volcano and seismicity from 17 July 2011 to 10 October 2011.

$$y(i) = \sum_{k=1}^{i} \left[x(k) - x_{\mathrm{ave}} \right].$$

The integrated time series is divided into N_S=int(N/s) non overlapping segments of equal length s (called time scale), starting from the beginning of the series. Because N/s often is not an integer, a short part at the end of the profile y(i) may remain. To not disregard such part, the same procedure is repeated starting from the opposite end. Thereby, $2N_S$ segments are obtained altogether. The local trend for each of the $2N_S$ segments is calculated by a least square fit of the series. Afterwards, the variance is computed

$$F^2(s,\nu)=\frac{1}{s}\sum_{i=1}^{s}\{y[(\nu-1)s+i]-y_\nu(i)\}^2$$

for each segment ν, $\nu=1,..,N_S$ and

$$F^2(s,\nu)=\frac{1}{s}\sum_{i=1}^{s}\{y[N-(\nu-N_S)s+i]-y_\nu(i)\}^2$$

for $\nu=N_S+1,..,2N_S$, where $y_\nu(i)$ represents the p-th degree fitting polynomial in segment ν. Averaging over all segments the following q-th order fluctuation function

$$F_q(s)=\left\{\frac{1}{2N_S}\sum_{\nu=1}^{2N_S}\left[F^2(s,\nu)\right]^{\frac{q}{2}}\right\}^{\frac{1}{q}}$$

is obtained, where, in general, the index variable q can take any real value except zero.

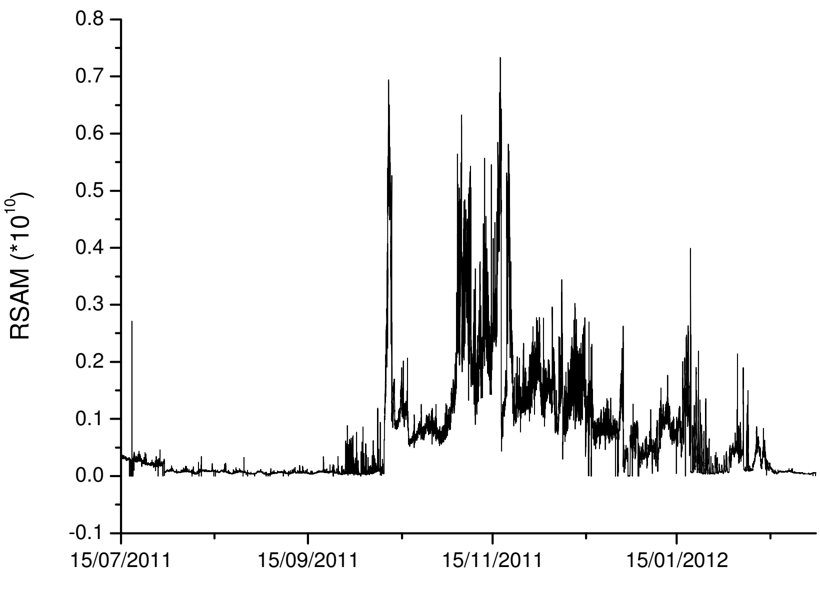

Fig. 2. RSAM data measured at El-Hierro volcano from 15 July 2011 to February 2012.

Repeating the procedure described above, for several time scales s, $F_q(s)$ will increase with increasing s. Then analyzing log-log plots $F_q(s)$ versus s for each value of q, the scaling behaviour of the fluctuation functions can be determined. If the series x_i is long-range power-law correlated, $F_q(s)$ increases for large values of s as a power-law

$$F_q(s) \propto s^{h(q)}.$$

The value h(0) corresponds to the limit h(q) for $q \rightarrow 0$, and cannot be determined directly using the averaging procedure above because of the diverging exponent. Instead, a logarithmic averaging procedure has to be employed,

$$F_0(s) \equiv \exp\left\{ \frac{1}{4N_S} \sum_{\nu=1}^{2N_S} \ln\left[F^2(s,\nu)\right] \right\} \approx s^{h(0)}.$$

In general the exponent h(q) will depend on q. In particular for monofractal series it is independent of q. For stationary time series, h(2) is the well-defined Hurst exponent H (Feder, 1988). Thus, we call h(q) the generalized Hurst exponent. The different scaling of small and large fluctuations will yield a significant dependence of h(q) on q. For positive q, the segments ν with large variance (i.e. large deviation from the corresponding fit) will dominate the average $F_q(s)$. Therefore, if q is positive, h(q) describes the scaling behaviour of the segments with large fluctuations; and generally, large fluctuations are characterized by a smaller scaling exponent h(q) for multifractal time series. For negative q, the segments ν with small variance will dominate the average $F_q(s)$. Thus, for negative q values, the scaling exponent h(q) describes the scaling behaviour of segments with small fluctuations, usually characterized by larger scaling exponents.

The multifractal scaling exponents h(q) are directly related to the scaling exponents τ(q) defined by the standard partition function multifractal formalism (Kantelhardt et al., 2002)

$$\tau(q)=qh(q)-1.$$

The singularity spectrum f(α) is related to τ(q) by means of the Legendre transform (Parisi and Frish, 1985),

$$\alpha = \frac{d\tau}{dq}$$

$$f(\alpha)=q\alpha-\tau(q),$$

where α is the Hölder exponent and $f(\alpha)$ indicates the dimension of the subset of the series that is characterized by α. The singularity spectrum quantifies in details the long-range correlation properties of a time series.

2.1.2 The results

We divided the whole observation period in three windows:

a) from 15 July 2011 to 10 October 2011 (pre-eruptive phase)

b) from 10 October 2011 to 22 November 2011 (1^{st} eruptive episode)

c) from 22 November 2011 to 29 February 2012 (2^{nd} eruptive episode)

Fig. 3 shows the multifractal spectrum of the whole RSAM record along with those of the three data segments, as defined above.

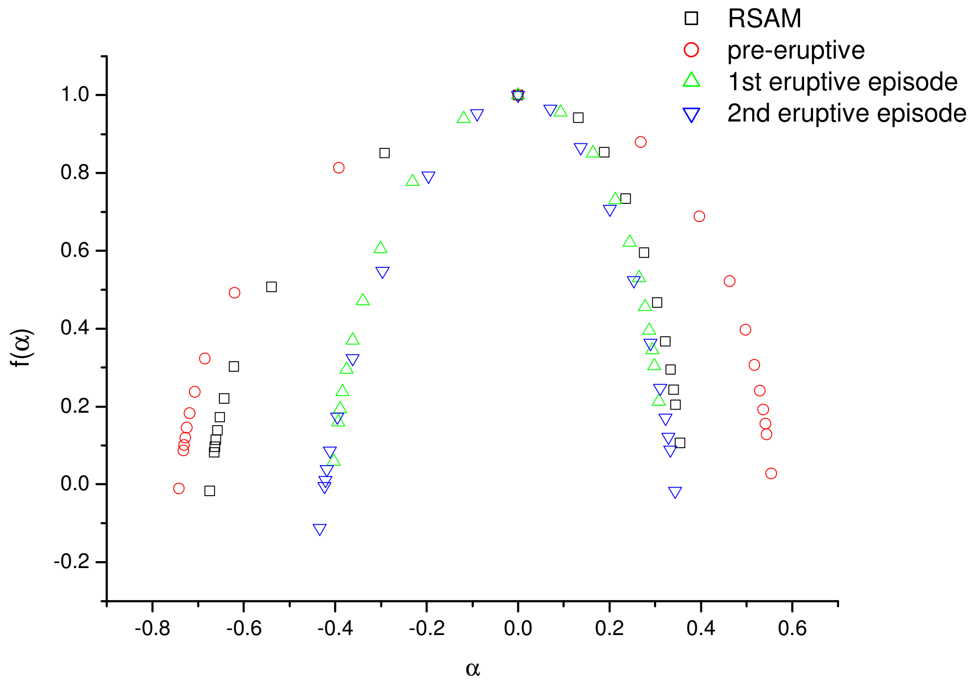

Fig. 3. Multifractal spectra of RSAM and the three data segments as defined in the text.

The multifractality can be quantified by means of the width, defined as $\Delta=\alpha_{max}-\alpha_{min}$,

and the asymmetry, given by $A=\dfrac{\alpha_0-\alpha_{min}}{\alpha_{max}-\alpha_0}$, where α_0 is the α-value at which f(α) is maximum, and α_{min} and α_{max} are the minimum and the maximum α-value. A more or less multifractal signal (which means that Δ is large or small), implies a more or less heterogeneous signal; a signal is heterogeneous if it is characterized by sudden bursts of high frequency, intermittency, irregularity. The asymmetry indicates the relative dominance of the low and high generalized Hurst exponents, implying the relative dominance of the large and small fluctuations respectively; A=1, the spectrum is symmetric; if A>1, the multifractal spectrum is left-skewed; if A<1 the spectrum is right-skewed. A left-skewed spectrum indicates that the dynamics of the signal is dominated by the large fluctuations, while a right-skewed spectrum indicates that the small fluctuations dominate the dynamics of the signal. In our case RSAM data have Δ~1.03 and A~1.9. These values suggest that the RSAM data are characterized by a certain multifractality degree, meaning a certain degree of heterogeneity; furthermore, the dynamics is dominated by the relative large fluctuations.

	Δ	A
RSAM	1.03	1.9
Pre-eruptive	1.296	1.338
1st eruptive episode	0.711	1.312
2nd eruptive episode	0.778	1.263

Table 1. Multifractal characteristics of the data

Comparing these values with those obtained for the three different data segment (see Table 1), Δ and A for the RSAM can be considered a sort of average among the corresponding values of three segments. It is striking that the pre-eruptive phase has a value of the multifractal width significantly larger than that of the two successive eruptive episodes. However, all the three spectra have very close values of the asymmetry. These results indicate that during the pre-eruptive phase the seismic signal is characterized by a larger heterogeneity and irregularity, even though in all the three phases the dynamics is dominated by the relative large fluctuations. Furthermore, it is also striking the quasi-collapsing of the two multifractal spectra of the signal segments recorded during the two eruptive episodes, indicating similar multifractal fluctuations influencing the dynamics of the signal in these two eruptive episodes.

2.2 Entropic-informational analysis

2.2.1 The method

The complex temporal fluctuations of nonstationary signals can be well investigated by using two informational measure: the Fisher Information Measure (FIM) and the Shannon entropy power (N_X), well known in the information theory framework. The FIM quantifies the amount of organization or order in a system, while N_X measures its degree of uncertainty or disorder. The Shannon entropy quantifies the amount of uncertainty of the prediction of the outcome of a probabilistic event, being zero for deterministic events. Therefore, higher the Shannon entropy of a system, higher its degree of unpredictability.

Formally, both the FIM and the Shannon entropy are defined as follows. Let *f(x)* be the probability density of a signal x, then its FIM I is given by

$$I = \int_{-\infty}^{+\infty} \left(\frac{\partial}{\partial x} f(x) \right)^2 \frac{dx}{f(x)},$$

and its Shannon entropy is defined as:

$$H_X = -\int_{-\infty}^{+\infty} f_X(x) \log f_X(x) dx .$$

For continuous distributions the Shannon entropy can take any real positive and negative value. Therefore, in order to avoid to deal with negative measures, the so-called Shannon power entropy N_X can be used instead of the Shannon entropy:

$$N_X = \frac{1}{2\pi e} e^{2H_X} .$$

The calculation of the FIM and the Shannon entropy power depends on the reliable estimation of the probability density function *f(x)* (pdf), which can be performed by means of the kernel density estimator technique (Devroye, 1987; Janicki and Weron, 1994) that approximates the pdf as:

$$\hat{f}_M(x) = \frac{1}{Mb} \sum_{i=1}^{M} K\left(\frac{x - x_i}{b}\right),$$

with b the bandwidth, M the number of data and $K(u)$ the kernel function, which is a continuous non-negative and symmetric function satisfying the two following constraints:

$K(u) \geq 0$ and $\int_{-\infty}^{+\infty} K(u)du = 1$.

In our study, the estimation of the pdf was performed by using the algorithm developed in Troudi et al. (2008) combined with that developed in Raykar and Duraiswami (2006), that uses a Gaussian kernel with zero mean and unit variance:

$$\hat{f}_M(x) = \frac{1}{M\sqrt{2\pi b^2}} \sum_{i=1}^{M} e^{-\frac{(x-x_i)^2}{2b^2}} .$$

2.2.2 The results

The time-varying FIM and N_X was investigated for the RSAM signal measured at El-Hierro. A sliding time window was employed to calculate the time variation of the local informational parameters. A sliding window of 10^3 samples (~7 days) was used to compute the local FIM and N_X. Each computed value was associated with the time of the last sample in the sliding window. The shift between two successive windows was set at 10 samples (equivalent to 10^2 minutes), in order to smooth the results and evaluate the variation of the informational parameters with a relatively high time resolution. Fig. 4 shows the results of the local FIM.

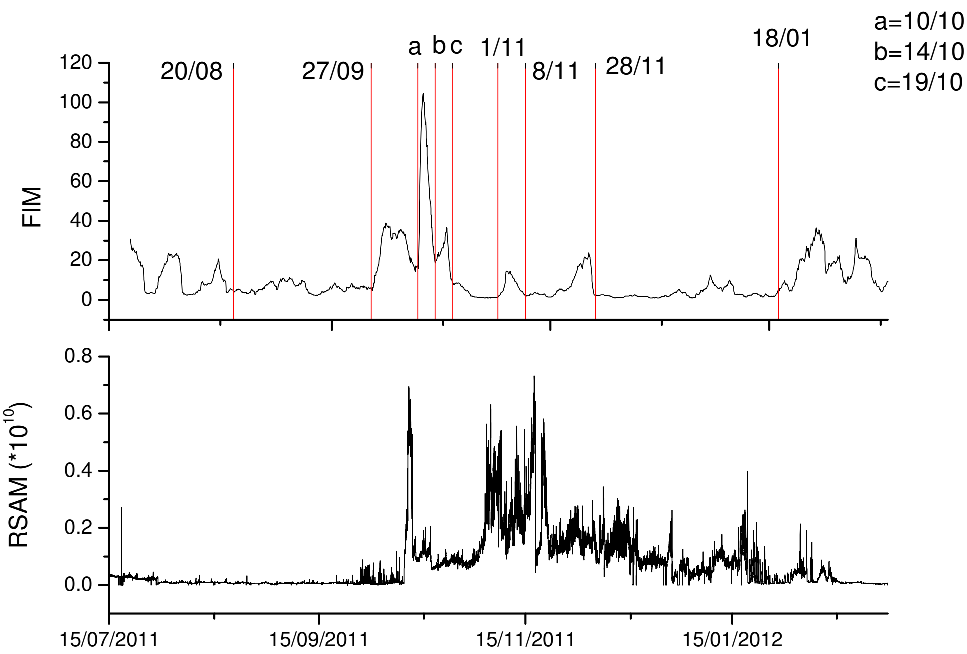

Fig. 4. Time-varying FIM and RSAM data.

Generally speaking, the variability of the local FIM indicates that the volcanic system of El-Hierro changes its dynamical state between less organized structures (low FIM values) and more organized ones (high FIM values). A more detailed analysis of the FIM variation shows several features that are not directly observable in the RSAM time series: i) from the beginning of the recording until 20 August 2011, the local FIM is characterized by an oscillating behavior; ii) then, until approximately 27 September 2011, the local FIM is quite stable with relatively low values; iii) during the successive period until the onset of the eruption on 10 October 2011, the local FIM is characterized by a visible increase that culminates in the highest FIM value about 1 days after the onset of the eruption on 11 October 2011, followed by a smaller peak on 18 October 2011, but dropping down almost suddenly on 19 October 2011; iv) from late October 2011 until late November 2011, the FIM shows the presence of other two visible peaks, about at 3 and 26 November 2011; v) then it stabilizes at relatively low values until 18 January 2012, since when it starts to have a quasi-oscillating behavior similar to that shown at the beginning of the record.

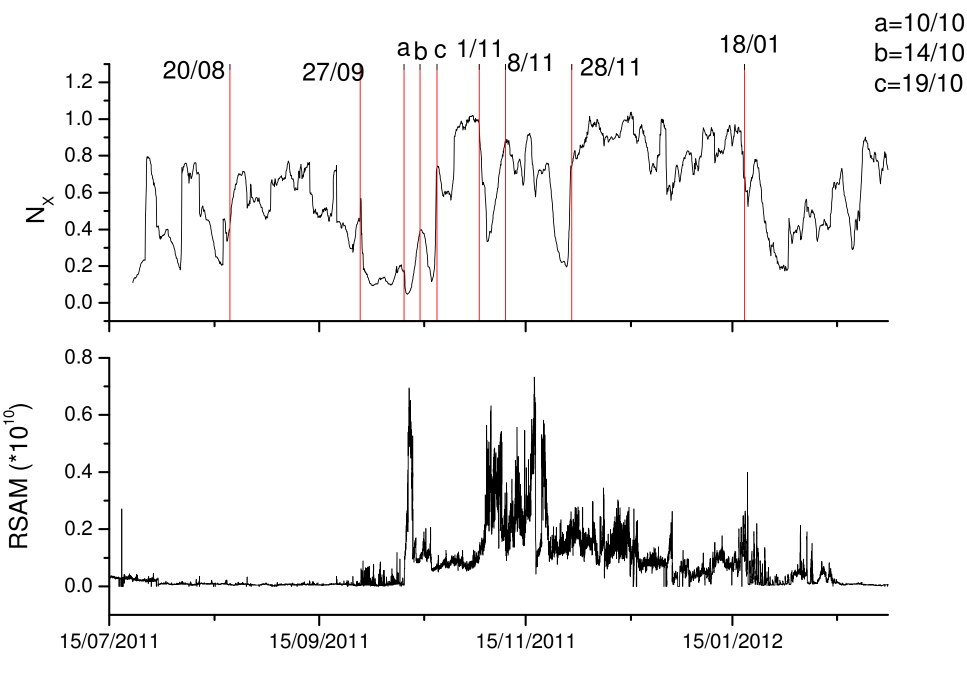

Fig. 5. Time-varying N_X and RSAM data.

Fig. 5 shows the time variation of the Shannon entropy power N_X. The Shannon entropy power quantifies the amount of disorder or uncertainty of a system, so it behaves approximately in opposition with the FIM. Although the behavior of the

N_X appears more irregular than FIM, it is still recognizable the main pattern as we detected in the FIM time variation: drops (which have a counterpart in the FIM peaks), and the quasi-stable phases (which correspond to the quasi-stable phases in the FIM evolution). However, it is clear that, of the two, the FIM is a much better indicator of the underlying dynamical change in the volcanic system of El-Hierro. In fact, the amplitude of the FIM peaks is larger than that of the N_X drops, indicating that the FIM yields more discrimination power between the different states of the volcano.

3. CRITICAL EVALUATION OF THE PROJECT

The research activity performed during my stay at Instituto de Ciencias de la Tierra Jaume Almera (CSIC) was successful and the objectives of the STM project were fully reached.

In particular:

1) It was planned to submit two papers at scientific International journals with the citation of the STM 2013 CNR Program;

2) The scientific collaboration was strengthened not only with the ICTJA-CSIC in Barcelona, but also with the IGN in Madrid

4. ACKNOWLEDGEMENTS

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