Sort-term Mobility 2010 Report

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Introduction

This document is the final report about research activities in the field of adaptive medical robotics at Mechatronics In Medicine (MIM) laboratory, Dept. of Mechanical Engineering, Imperial College London (UK), from July 3 to July 31, 2010, within *Short-Term Mobility* (STM) Program granted to Federico Vicentini, proposed by Lorenzo Molinari Tosatti, both in service at Institute of Industrial Technologies and Automation (ITIA).
The research topic involved the development of a robotic platform for the bio-mechanical analysis of the human knee articulation. This research program at MIM was at its very early stage at the moment of the STM period, then some preliminary activities were expected in the development of the experimental setup, mainly related to the identification of the key figures for the application of any adaptive control. The overall objective of the research program is in fact the investigation about robot learning and adaptation w.r.t. the changing and partially observable conditions of a target system. The peculiarity of the learning is due to the fact that the robot must understand the behavior of the target system (knee articulation) from the mutual interaction, i.e. through an exchange of forces depending on the position and velocity of such interaction.

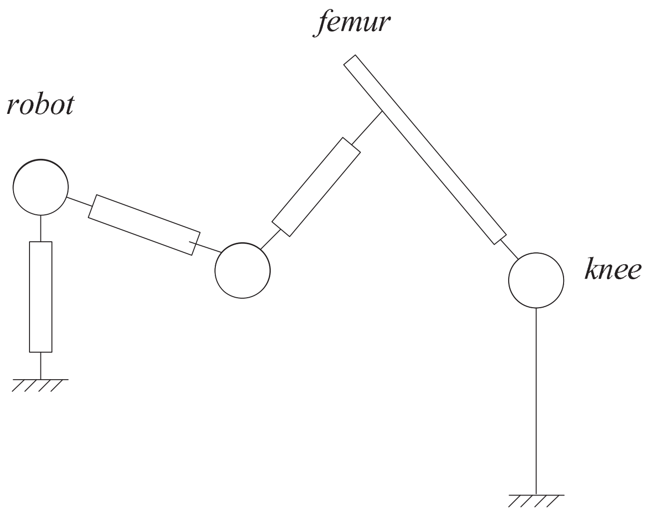

Figure 1: Schematic representation of the interaction between the robot (on the left) and the bio-mechanical target system (on the right). The target system is rigidly attached to the robot by a gripper mounted on the end effector after a force/torque sensor.

In addition, the accuracy of the estimated mutual position plays a key role in the control process since the variation in the response of the target system could display a little relative magnitude in forces and very little usable workspace on the target system (the nature and extension of the mechanical properties of the articulation under investigation).
For these reasons the research activities focus on the identification of robot dynamics and the accuracy of system calibration.

System setup and identification issues

The experimental setup includes a Staubli TX90 robot rigidly attached to a femur/ligaments specimen for the knee joint analysis. Such analysis entails the mechanical properties of the articulation. In particular the key feature to be investigated is the detection of structural flaws in bones and/or tendons. Feedback signals are provided by the robot force sensor at the TCP and by additional sensors directly on the specimen. The robot plays therefore a key role in the excitation of trajectories suitable for the detection of potential flaws in the biomaterials as a function of the force response. Hence, an adaptive control approach is desired for optimally setting in real time the robot trajectories and force profiles. In particular, given the setup in Fig. 1, let Bio-Mechanical System (BMS) be the limb specimen that is displaced in the Cartesian space by the robot trajectory $\mathbf{x}_r$ as in Fig. 2; let the TX90 and its controller (CTRL) be the robotic platform forcing the BMS given the control variables $\mathbf{u}$ or the joint coordinates $\mathbf{q}$ to follow. Then, the trajectory of the BMS is tracked and supplied to a Trajectory Planner (TP) that is in charge of computing the runtime setpoint $\mathbf{x}_r^{\mathbf{0}}$ for the robot in the Cartesian space. The Inverse Kinematic (IK) block is inserted for consistency.

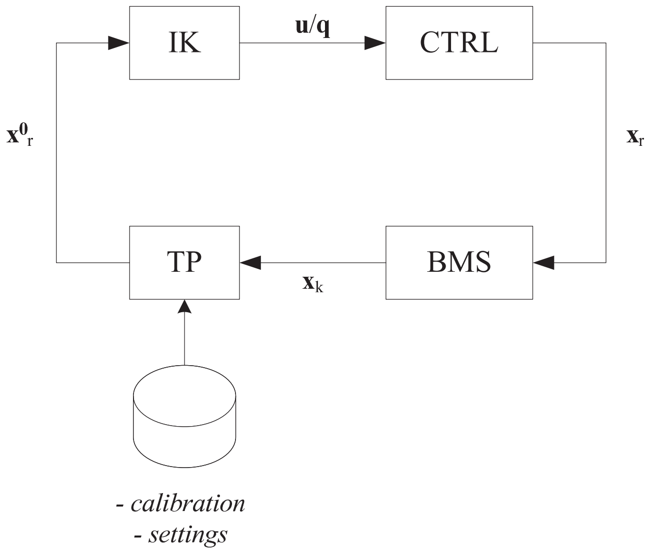

Figure 2: Functional blocks involved in the control loop of the robot-limb system. IK = Inverse Kinematics, CTRL = robot and controller, BMS = target limb, TP = trajectory planner

The TP is in charge of *understanding* the current conditions of the BMS in order to set/modify the excitation trajectories. Hence it makes use of runtime machine learning routines for the estimation of BMS state and generation of $\mathbf{x}_r^{\mathbf{0}}$, on the basis of a pre-loaded knowledge base made of *control settings* and *calibration* parameters.

In next sections the issues related to the robot dynamical parameters identification for optimally set the control strategy and the system calibration are discussed.

Settings

Among the direct/indirect methods for identification of robot dynamical parameters ([1]), most apply for widely known laboratory robots, like the Staubli PUMA (*e.g.* [2, 3]) and RX series ([4]) or Mitsubishi PA-10 ([5, 6]) because the direct access to the low level control. Most of industrial commercial robot do not allow real-time access to control routines. In such cases, the motor torque can only be recorded during execution time by asynchronous query routines. Hence, being the robot in control and under the hypothesis of fast execution of the sampling routine, the sampled data introduce estimation errors due to the indirect access to the controller. This is the case of the Staubli TX90. Nevertheless, estimation can take place [7]. However, the dynamic parameters under identification requires to be clustered during the computational process and, most importantly, an explicit model of the type

$$\mathbf{\Gamma}(\tau) = \mathbf{W}(\theta, \dot{\theta}, \ddot{\theta})\mathbf{p}$$

must be used for the computation of a least square solution for the unknown parameters $\mathbf{p}$ in terms of

$$\mathbf{p} = \mathbf{W}^{+}\mathbf{\Gamma}$$

where θ, $\dot{\theta}$, $\ddot{\theta}$ are the joints coordinates, $\mathbf{W}$ is the equivalent matrix of all inertial, frictional, centripetal and gravitational contributions to the dynamical model, $^{+}$ denotes the Moore-Penros pseudo-inverse, $\mathbf{\Gamma}$ is the array of joint torques at given joint positions. Such explicit representation of $\mathbf{p}$ unknowns go through a non-trivial manipulation of the basic dynamical model

$$\tau = \mathbf{M}(\theta, \mathbf{p})\ddot{\theta} + \mathbf{C}(\theta, \dot{\theta}, \mathbf{p}) + \mathbf{F}(\theta, \mathbf{p})\dot{\theta} + \mathbf{G}(\theta, \mathbf{p}) \qquad (1)$$

for any time step. In particular, under the Lagrange formulation of (1), the representation of $\mathbf{M}$, $\mathbf{C}$, $\mathbf{F}$ and $\mathbf{G}$ requires the sum of each link contribution, hence a sum of matrices multiplication. As a result, this approach is usually followed by robot manufacturers with very limited public availability of results (mainly used in calibration), and rarely solved in analytical or, rather, numerical way.

As an alternative methodology, several approaches involve artificial neural networks for control [8] and parameters identification [9], [10] and *op. cit.*. Jiang et al. (2006) [11] introduced a neural dynamic compensator of parameter observation where the learning algorithm plays a key role in the fine tuning of the system identification. Kosmatopoulos et al. (1995) [12] instead envisaged the use of second-order Hopfield networks for the approximation of an observed dynamical system through a learning dynamic modeler, where the model parameters are the unknown and several trajectories are sampled. In this case the input is an array of given motor torques that provide a joint position pattern. In simulation this can be achieved applying the direct dynamics to the robot model, while in real applications the torque control on given torque pattern is unusual. Rather, considering that the robot is of course controlled during any data acquisition routine, the wise way is to define a set of trajectories suitable for good excitation ([13]) of all the dynamic parameters together with good computational conditions, i.e. stable patterns of torques. Therefore, good trajectories involve steady rate of all the joints, axis-by-axis jog motions and Cartesian directions of motion non-parallel w.r.t. the projections of links principal inertial axes.

Hand-Eye calibration

Given the system setup of Fig. 1, the limb specimen requires to be tracked in order to assess the kinematics profiles w.r.t. the robot coordinate frame. With reference to Fig. 3, $\{r\}$ is the robot base coordinate frame, while $\{h\}$ is the end effector moving coordinate frame. $\mathbf{T}_{\{rh\}|i}$[1] is the homogeneous transform for the roto-translation of the end effector, *i.e.* the direct kinematics of the robot. The target BMS motion is tracked by a marker-based capturing system (*i.e.* NDI Polaris): tracked markers are represented by a cluster of points in the $\{e\}$ coordinate frame, while $\{c\}$ is the base coordinate frame of the capturing system. Let therefore be $\mathbf{Z}$ and $\mathbf{Z}_k$ the roto-translations of the base coordinate frames of the robot and of the specimen from the $\{c\}$ frame, respectively. The tracking markers provide a corresponding roto-translation $\mathbf{T}_{\{ce\}|i}$ of the tracked limb (*i.e.* femur) w.r.t. the $\{c\}$ frame. The robot grasps the specimen on a generic

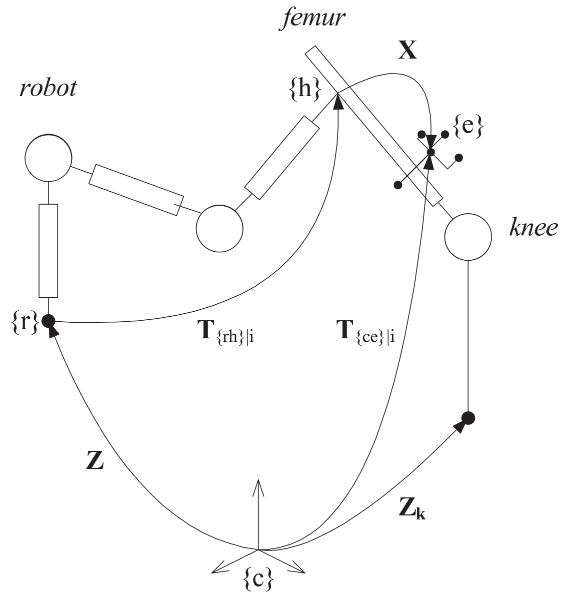

Figure 3: Notation of hand-eye calibration frames and transforms.

point of the specimen, whose position is non-trivially measured but requires calibration. Such calibration is represented by the rigid constant transform $\mathbf{X}$ from $\{h\}$ to $\{e\}$ frames. It is usually known as *Hand-Eye Calibration Problem.*

The hand-eye problem was originally posed more than two decades ago by Shiu and Ahmad in their seminal work [14], stemming from the calibration problem of robot-mounted sensors, shortly followed by Tsai and Lenz [15] who properly formulated the kinematic framework and the geometrical constraints affecting the accuracy of computation. The problem in fact entails the identification of the unknown transform between the robot end effector frame and the sensor/tool carried/tracked on board of the gripper. Knowledge of such transform allows any employed control model to directly feedback the carried

[1] The subscripts in $\mathbf{x}_{\{fg\}|i}$ are: f/g the initial/final coordinate frames from/to where the matrix $\mathbf{x}$ is referred, i the pose of the robot during a trajectory made of $i = 1, \ldots, N$ poses.

sensor/tool position. Since those early works, the number of applications requiring a calibration of the tools/sensors carried by robot grippers has greatly expanded. Notable examples include, among others, visual servo control ([16, 17], [18] and *op. cit.*,) and medical robotics ([19, 20, 21, 22, 23]). Remarkably in these latter uses, the accuracy of the calibration is a key outcome for the whole control process, namely when a fine positioning of tools w.r.t. a surgical targets is required. Another class of application involves the tracking of reflective markers clusters (*eyes*) attached to any link (*hand*) of the robot or attached directly on tools possibly carried by hand, *e.g.* surgical endoscopes, like in [24]. Markerless motion detection techniques are included as well for *eye* and/or *hand* movements capture. Ultimately, the common framework involves two ways of capturing the same motion of a fiducial object.

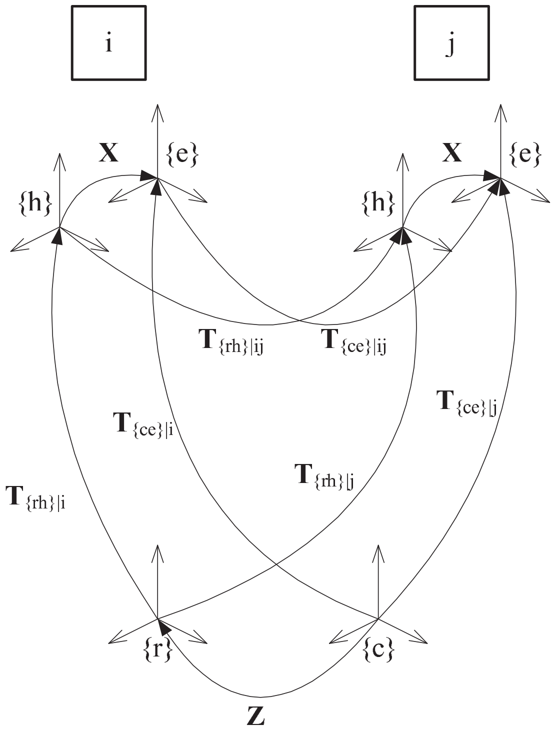

Figure 4: Hand-Eye notation. i and j represent two different poses.

Considering notation introduced above and displayed in Fig. 4,

$$\mathbf{Z}\mathbf{T}_{\{rh\}}\mathbf{X} = \mathbf{T}_{\{ce\}} \tag{2}$$

expresses the closed kinematic loop among the coordinate frames. Considering also N different robot poses, two relations stem from (2) for any $i = 1, \ldots, N$:

$$\begin{aligned} \mathbf{T}_{\{rh\}|i}\mathbf{X} &= \mathbf{Z}^{-1}\mathbf{T}_{\{ce\}|i}, \\ \mathbf{T}_{\{rh\}|ij}\mathbf{X} &= \mathbf{X}\mathbf{T}_{\{ce\}|ij} \quad i \neq j,\ ij = ji. \end{aligned}$$

that are known since early literature works as:

$$\mathbf{A}_i\,\mathbf{X} = \mathbf{Z}\,\mathbf{B}_i, \tag{3}$$

$$\mathbf{A}_{ij}\,\mathbf{X} = \mathbf{X}\,\mathbf{B}_{ij}. \tag{4}$$

where

$$\mathbf{A}_i = \mathbf{T}_{\{rh\}|i} \qquad \mathbf{B}_i = \mathbf{T}_{\{ce\}|i}$$
$$\mathbf{A}_{ij} = \mathbf{T}^{-1}_{\{rh\}|i}\mathbf{T}_{\{rh\}|j} \qquad \mathbf{B}_{ij} = \mathbf{T}^{-1}_{\{ce\}|i}\mathbf{T}_{\{ce\}|j}.$$

The formulation in (4) displays a relationship between (i, j) pairs of poses, hence a quadratic equation in terms of relative poses. It is also far more common in early literature and allows a geometric interpretation of $AX = XB$ problem that underlies the solution approach of many of the algorithms developed in last two decades, like [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36]: $\mathbf{A}$ and $\mathbf{B}$ are the same rototranslation projected in two different coordinate frames ($\{h\}$ and $\{e\}$) whose relative pose is represented by $\mathbf{X}$, as shown in [37, 26]. Because most of practical applications (*e.g.* control, visual guidance, tracking, etc) make use of calibration matrix $\mathbf{X}$ for aligning $\mathbf{A}$ and $\mathbf{B}$ in a common coordinate frame, an alternative interpretation shows $\mathbf{A}$ and $\mathbf{B}$ as two rototranslations of the same magnitude around two skew screw axes [37]. Works in [38, 39, 40] provide instead a solution for the problem posed in terms of $AX = ZB$.

More in detail, Tsai and Lenz (1989) formulated in [41] a variation of the axis/angle notation for rotations in order to be linearized, exploiting in this way the similarity property of (4) in rotational part, *i.e.* $\mathbf{R}_A = \mathbf{R}_X\mathbf{R}_B\mathbf{R}_X^T$. From this property, the meaning of $\mathbf{X}$ is the transform that rotates the axis of $\mathbf{R}_A$ to overlap that of $\mathbf{R}_B$. Linearized system searches for the least square solution for such transform. Park and Martin (1994) introduced in [30] the use of Lie algebra in order to linearize the equation system originating from (4). The physical consistency holds because Lie algebra is equivalent to the analysis of the first order kinematic. The rotational and translational parts of solution $\mathbf{X}$ are separately treated in the closed-form noise-free solution. Despite many recent works assume that errors in $\mathbf{R}_X$ propagate through $\mathbf{t}_X$ if separately solved, authors demonstrate that, in the Lie algebra, the minimization of rotational part through a least square solution minimizes also the translational contribution. This is due to the fact that, in the Lie algebra, the distance between rototranslations can be expressed as linear function of the error in translation and the error in rotation (rotation linearization). Daniilidis (1999) improves in [33] the use of dual quaternions as mathematical tool for representing generic rototranslation, extending the discussion of [42]. The formulation allows the merge of both rotation and translation in only one equation. Despite the very elegant solution (see also [43]), the dual quaternion formalization involves the algebra of dual numbers, dual vectors and dual quaternion that is not completely addressed in the implemented algorithm. The closed-form solution in fact relies on separated scalar and dual parts of dual entities, preventing the full exploitation of the underlying algebra. Nonetheless, the formulation is extremely compact and the over-constrained problem of a large number N of poses is solved through the kernel of a $6N \times 8$ matrix via a SVD routine. Andreff, et. al (1999) first introduced ([34], often referred as [44]) a linear formulation of the hand-eye problem including translation with rotation at the same time. Such formulation features the common usage of equation (4) in control theory as a variant (as it is) of the Sylvester equation[2], with the additional constraint of dealing

[2] $\mathbf{UV} - \mathbf{VW} = \mathbf{T}$ where $\mathbf{U} = \mathbf{A}$, $\mathbf{W} = -\mathbf{B}$, $\mathbf{V} = \mathbf{X}$ and $\mathbf{T} = \mathbf{0}$

with homogeneous matrices. Linear formulation exploit the Kronecker product[3] operator. However the linear formulation does not guarantee that the found solution belong to $\mathcal{SE}(3)$[4], yet rotation potentially requires to be orthogonalized. Angeles, et. al (2000) in [35] approaches the problem by separating $\mathbf{R}_T$ from $\mathbf{t}_T$. Nevertheless, like in [30], the linearization of the problem is performed within the first order kinematic, *i.e.* the minimum found for rotational part minimizes also the errors in translational contribution. The most interesting feature consists on the implementation of a recursive procedure that incrementally refines the accuracy of calibration until no further progress can be achieved. Furthermore, authors suggest a very interesting methods for the optimization of iterative computation of the pseudo-inverse matrix. Despite these very interesting aspects, the basic idea is still the computation of a least square solution w.r.t. the population of samples. Dornaika and Horaud (1998) first introduced in [31, 39] the non-linear minimization of the error in (3) in terms of Frobenius norm of $\mathbf{A}\hat{\mathbf{X}} - \hat{\mathbf{X}}\widetilde{\mathbf{B}}$, where $\hat{\mathbf{x}}$ denotes an estimate of $\mathbf{x}$ and $\widetilde{\mathbf{x}}$ denotes a noisy observation of $\mathbf{x}$. Despite the loss of generality in the difference of homogeneous matrices, the algorithm features the searching of a global minimum for the convex error function with weighted contributions of rotation/translation errors and of the constraint of orthogonality, therefore not requiring any orthogonalization. Strobl and Hirzinger (2006) mainly approach the$AX = ZB$ problem, although analysis and implementation are suitable for the $AX = XB$ problem too. Authors first introduced in [40] a mitigation factor for the impact of dimensionality in rotation and translation components making use of a weighted $\mathcal{SE}(3)$ metric for rototranslations. A key point of the algorithm is the correction of erroneous measurements of observed data using the samples noise model and coupling the estimation and prediction in the same metric for the purpose of optimization, and it estimates $\mathbf{X}$ and $\mathbf{Z}$ by minimizing the squared prediction error in the estimate $\mathbf{A}\hat{\mathbf{X}} = \hat{\mathbf{Z}}\mathbf{B}$ of solution for (3). From a mathematical point of view, it is an actual application of the maximum likelihood (ML) method that select the models $\hat{\mathbf{X}}$ and $\hat{\mathbf{Z}}$ with the highest probability to fit the observed data.

All the reported algorithms seek for accuracy in the estimation of unknown calibration matrices $\mathbf{X}$ and $\mathbf{Z}$, defined as the bias w.r.t. the nominal value. Such bias and need of estimation is introduced by the measurement noise that all the times occurs when robot poses are tracked by the capturing system. The solution is therefore estimated minimizing the expected error of $\mathbf{X}$ among the set of observed data. Multiple measurements are required ($N \geq 2$ poses) and solutions are provided in terms of either least square or nonlinear minimization of (3) or (4). An intrinsic limit in achieving the maximum accuracy is however posed by the minimization strategies among many noisy data that are variously proposed in literature. Despite different formulations, the core geometrical setup holds. Thus the accuracy of the estimation depend on the distribution of noise in observed data.

[3] $(\mathbf{A} \otimes \mathbf{I} - \mathbf{I} \otimes \mathbf{B})vec(\mathbf{X}) = vec(\mathbf{0})$

[4] $\mathcal{SE}(3)$ is the Euclidean group of rigid body motions, which rototranslations $\mathbf{T}$ belong to. $\mathbf{R}_T \in \mathcal{SO}(3)$, *i.e.* $\mathbf{R}_T$ is a 3×3 matrix representation of an element in the $\mathcal{SO}(3)$ Special Orthogonal group of rotations, $\mathbf{t}_T \in \mathbb{R}^3$.

Conclusions and Future Works

Some tasks are required at the early stage of the development of the robotic platform for the analysis of biomaterials through force interaction. The most relevant ones include the estimation of dynamical model of the robot and the reference calibration of the system. The estimation of dynamical parameters for the manipulator have direct impact on the control strategies performed by the robot because adaptive procedures are usually based on the accurate knowledge of the agent/system under adaptation. In particular, the robot control is inc charge of setting in real time the characteristics of the trajectories and force profiles that are intended for the excitation of the target specimen. Hence the robot has likely to forecast the behavior of the interaction between the target system and itself along the contact directions. The assessment about the expected behavior of the interaction is closely related to an accurate dynamical model in the case of model-based control of the manipulator. Alternative control methodologies involve the driving system (*i.e.* the robot) as a black or gray box without any analytical model available. These methodologies however involve an additional layer of learning, possibly hindering the stability or efficiency of estimation results. Since most of identification techniques are either not available or analytically/computationally ineffective. Hence the application of second-order neural estimators for dynamical system is a key future activity. On the hand-eye calibration, all the algorithms developed in literature display a limit in improving the accuracy of the estimation of the calibration matrix. This limitation is due to the formulation of the problem in terms of quadratic relationship between same relative or absolute poses computed/measured from different coordinate frames. Hence the investigation and development of a new algorithm based on geometric properties of the system is a key future activity.

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