CNR Short term mobility — relazione finale: Statistical properties of Barkhausen avalanches in magnetic thin films

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Abstract

We discuss here the main results obtained at Cornell for our project on Barkhausen noise statistics in thin film. First we consider the analysis of avalanches obtained in a domain wall model, which should lead to a better characterization of magneto-optical measurement. Next we extract the pulse shape from the noise recorded in permalloy thin films and compare it with the theory. These results stem from a collaboration with J. P. Sethna, S. Papanikolau and Y. J. Chen from Cornell University.

1 Introduction

The jerky motion of domain walls in soft magnetic materials, a basic example of complexity in materials science, has been the subject of a long and continuing series of studies. In bulk magnetic systems, most of the statistical properties are now understood in terms of a depinning transition of the domain wall, and many experimental results are well explained in by these theories [1]. In thin films, the motion of walls is presumably dominated by depinning as well, but our understanding of the dynamics is still at a preliminary stage. Despite the lower dimensionality, it is well known that domain walls in two dimensions often exhibit more complex structures than simple Bloch walls, and the dominant interactions important for the wall dynamics are much more complicate to be determined. Also, from the experimental point of view, the data available are still limited, although the few recent papers published using the magneto-optical Kerr effect (MOKE) have shown interesting and promising results. In 2000, Puppin [2] measured the critical exponent τ of the avalanche size distribution $P(S) \sim S^{-\tau} f(S/S_o)$ in a Fe/MgO film of 90 nm to be about $\tau \sim 1.1$, much less than the values reported in the literature for bulk systems. Later, Kim *et al.* [3] indeed measured larger values $\tau \sim 1.3$ in Co thin films of different thicknesses (5 - 50 nm). Very recently, Ryu *et al.* [4] have argued that in a 50 nm MnAs film on GaAs(001), having a Curie temperature of only about 45 ^{o}C, there is a cross-over between different universality classes driven by temperature, with the exponent τ continuously changing from 1.32 to 1.04 as the temperature is increased from 20 to 35 ^{o}C. All of these experiments rely upon measuring avalanches optically, in a small sub-window of the entire sample. Usually, windows of varying sizes are used, and the distributions are superimposed to fit the critical exponent τ

over a few decades. In all these experiments, no information on the effect of the window size on the avalanche statistics is really taken into account, as, for instance, the dependence of the distribution cutoff S_o on the window sizes.

Here, we aim to explore some issues associated with extracting avalanche statistics from Barkhausen measurements. First we will discuss the role of the window size using a model for domain wall dynamics in disordered media. Next we consider the shape of Barkhausen pulses for inductive measurements in permalloy thin films.

2 Avalanches through windows

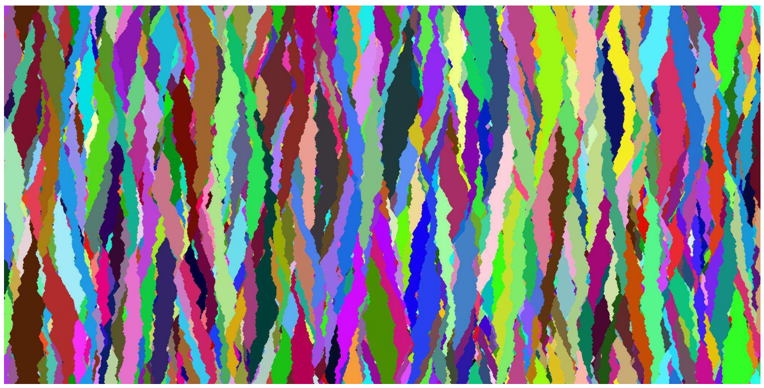

Figure 1: The avalanches recorded in the model. Here the x axis is along the vertical direction. Different avalanches are colored with random colors.

To explore the effect of windows size on the avalanche distributions, we consider a model for a domain wall moving in a random environment. We chose a model simple enough, allowing just the freedom to change the interface roughness as a function of one parameter. While the model in not completely realistic in term of the experiments, it provides a prototypical description of a self-affine interface moving in a disordered landscape. The equation of motion for the domain wall is given by

$$\frac{du(x,t)}{dt} = H - k\bar{u} + \nu\frac{d^2u}{dx^2} + \lambda\left(\frac{du}{dx}\right)^2 + \eta(x,u) \quad (1)$$

where $u(x,t)$ is the position of the domain wall,$\bar{h}$ is the center of mass of the wall that is proportional to the magnetization, $H(t)$ is a slow varying external field, k is the demagnetizing factor, $\nu\frac{d^2u}{dx^2}$ is the domain wall linear stiffness to which we added a small non-linear correction controlled by λ, and $\eta(x,u)$ is a random field describing all the inhomogeneities present in the sample. The effect of the non-linearity is crucial in two dimensions since otherwise the model would display anomalous scaling with super-rough behavior. We simulate a cellular automaton version of Eq. 1 where time and space are discretized and the local velocity can assume only the values 0 or 1. The model displays an avalanche distribution with scaling exponent $\tau = 1.18 \pm 0.01$. The domain wall is found to

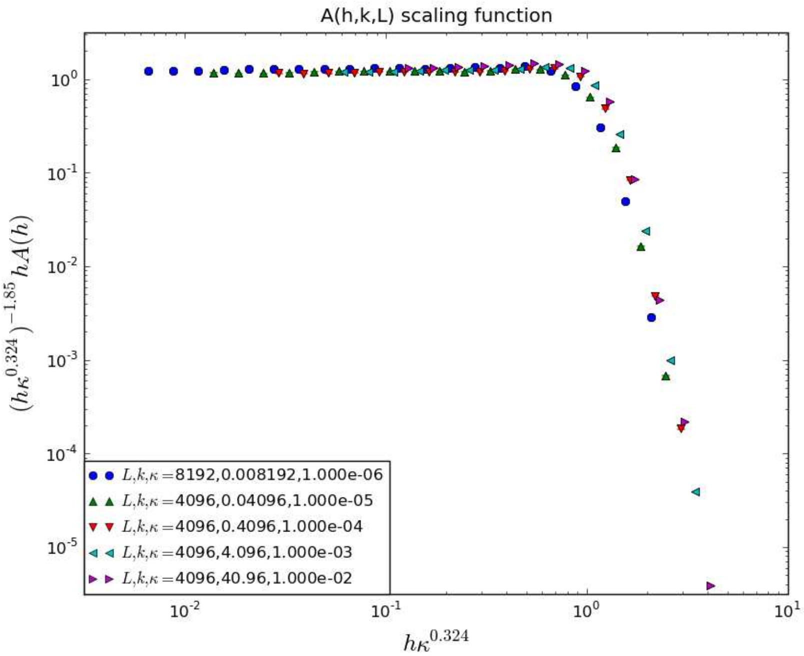

Figure 2: The data collapse of the area fraction of avalanches of width h.

be self-affine with a roughness exponent $\zeta = 0.63 \pm 0.05$. The model is simulated for system sizes $L = 4096, 8192, 16384$ and different values of $\kappa = k/L$.

If Fig. 2 we report a typical avalanche pattern observed for the model. We first characterize the avalanche statistics considering avalanche areas s, heights h and widths w. In order to avoid normalization problems and contrary to previous studies, we consider the area fraction of the avalanches A, rather than the density distribution. As an example, in Fig. 2 we show the data collapse for the area fraction of avalanches of height h. We obtain similar data collapses for the widths and the sizes. These statistical properties are needed to evaluate the corrections due to the finite window size, since the statistics will be corrected due to avalanches that are truncated by the sides of the measuring box. Work is currently in progress to determine the best collapse by an automatic fitting procedure, where all the different curves are fitted by a single master curve with scale dependent parameters. A first attempt of such a fit is reported in Fig. 3.

3 Shape of the Barkhausen pulse

The average pulse shape has been recently proposed as a sharp tool to characterize Barkhausen noise [5]. In analogy with critical phenomena, it is expected that pulses of different durations can be rescaled on a universal function, which would only depend on general features of the physical process underlying the noise. This scenario is supported by the analysis of a variety of models, where pulse shapes are described by universal symmetric scaling functions. In experiments,

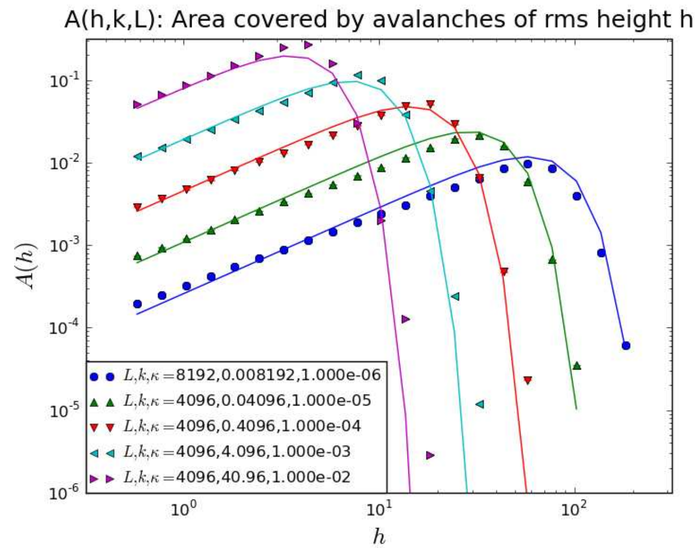

Figure 3: An automatic fit of the area fraction of avalanches of size h, for different κ and L.

however, the pulse shape is markedly asymmetric. These results are puzzling because the models accurately reproduce several other universal quantities, such as avalanche distributions and power spectra. The puzzled was recently resolved considering the effect of eddy currents [6]. These induce a delay in the avalanche response that creates an asymmetric response. Unfortunately there are no analytic solutions for the pulse shapes with eddy current and it is thus difficult to test the theory. For this it would be necessary to study Barkhausen noise *without* eddy current. Since eddy current depends on the sample thickness a possibility that we explore here is to consider thin films. First, we discuss a theoretical prediction for the shape and we then test it against experiments.

3.1 An analytic expression for the pulse shape

There have been two rival mean-field theories for Barkhausen noise in magnets – the single-degree of freedom ABBM model [7] and the infinite-range shell model [8]. The ABBM model treats the advancing domain wall as a rigid object at position $x(t)$, advancing in an external potential statistically chosen as a *random walk* in x:

$$\frac{dx}{dt} = c - kx + w(x) \tag{2}$$

with $\langle w(x)\rangle = 0$ and $\langle w(x)w(x')\rangle = |x - x'|$ and where c is the rate of increase of the external field $H(t)$ and k is a demagnetizing factor – which expresses the effect of free magnetic charges on the boundary of the sample, opposing the external field. The shell model has a set of interacting spins $M_i = \pm 1$ with random fields h_i distributed by a Gaussian, interacting with a strength J/N

with all other spins; each spin flips when the net field it feels

$$h_i + (H_0 + ct) + (J - k)/N \sum_j S_j \tag{3}$$

where $H(t) = H_0 + ct$ is the external field, increasing with rate c, and the spin feels the net magnetization $M = \sum_j S_j$ both through the long-range coupling J/N and the demagnetizing factor k. An avalanche proceeds in parallel, with a shell of V_n unstable spins flipping at time t_n, then triggering a new set of spins V_{n+1} to flip:

$$V_{n+1} = Ac + P(t, 2(J - k)V_n) \tag{4}$$

where P is the Poisson distribution for the set of spins V_{n+1} to be included in the range $\{f, f + 2(J - k)V_n\}$. A represents a constant that fixes the probability of having a random field in the regime $\{f, f + c\}$.

The average avalanche temporal shape in the ABBM model has not been extensively explored numerically, but an approximate analytical calculation [9] giving a lobe of a sinusoid $V(t/T) = T \sin(\pi t/T)$, hence predicting a universal scaling function $\mathcal{V}(\lambda) = \sin(\pi\lambda)$. The shell model had no analytical solution, but extensive numerical work clearly indicated that its scaling function was very close to an inverted parabola

$$\mathcal{V}(\lambda) = \lambda(1 - \lambda). \tag{5}$$

Upon closer examination, these two models are *the same* in the continuum limit. Bertotti [10] notes that the spatial noise can be transformed to a time - dependent noise at the expense of having a variance being proportional to the avalanche velocity V. This argument should become exact at the limit of small variance σ or large velocities V:

$$\frac{dV}{dt} = c - kV + \sqrt{V}\xi(t) \tag{6}$$

Similarly, in the shell model we may approximate the Poisson distribution as a Gaussian $P(V) = V + \sqrt{V}\xi$, leading to

$$\frac{dV}{dt} \simeq Ac - (k - J)V + \sqrt{(J - k)V}\xi(t) \tag{7}$$

which is clearly a discretized version of Bertotti's refomulated ABBM model 6, given that here there is a dependence of the noise's coefficient on k. However, since scaling is typically studied for k fixed (and in the limit $k \to 0$), this difference of the two equations is trivial. This equivalence, in retrospect, provides an alternative explanation for the somewhat mysterious choice of random walk statistics for the ABBM domain wall potential.

This equation is tangible enough to solve in the Stratonovich interpretation. We write the equation, after taking the continuum limit which describes the avalanche velocity:

$$\frac{dV(t)}{dt} = \sqrt{V(t)}\xi(t) \tag{8}$$

After defining a new variable $y = V^{1/2}$, we have a simple stochastic equation that we may solve explicitly:

$$\frac{dy}{dt} = \frac{1}{2}\xi(t) \tag{9}$$

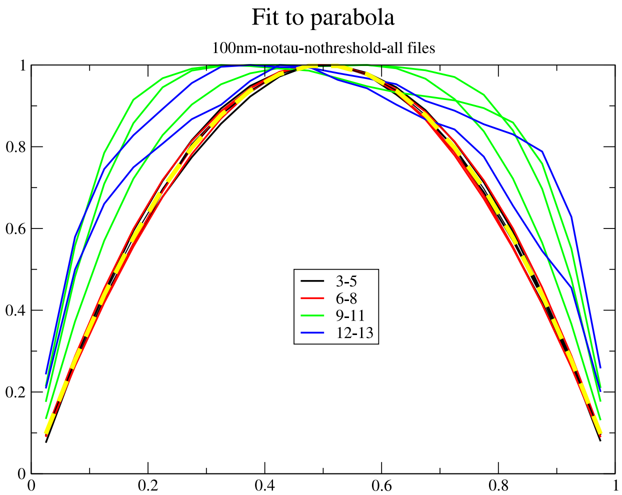

Figure 4: The rescaled average pulse shapes measured in a 100nm think permalloy thin film. The data for small duration are well described by a parabola.

The average temporal shape is defined by the first return of the random walker to the origin and the shape can be computed explicitly, and we find:

$$\langle V(t,\lambda)\rangle_p = \frac{1}{8}T\lambda(1-\lambda) \tag{10}$$

where $\lambda = t/T$ and the result holds at long times.

3.2 Experimental measurements

In order to test the theoretical predictions, we have analyzed Barkhausen noise measurements in permalloy thin films. The experiments were performed by F. Bohn and R. Sommer in Brazil. The main technical issue we faces in analyzing these data is the large background noise that makes the extraction of the pulse shape problematic. To overcome this problem, we use optimal Wiener filtering, estimating the noise spectrum from the background noise signal provided by the experimentalists. When this is done we can obtain an excellent signal to noise ratio and we can measure the avalanche shapes as a function of duration. The result reported in Fig. 4 is in excellent agreement with the theory at least for small durations. The deviations for larger duration can also be explained by the model, considering the effect of the demagnetizing factor.

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