Report on the visit to the Chemical Sciences Division of the NOAA Earth System Research Laboratory (7-28 October 2007) in the framework of the CNR Short Term Mobility Program

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During the course of the STM staying, I got accustomed to the theory and mathematical tools of Generalized Scale Invariance theory, and its application to the analysis of time series of geophysical fields. More specifically, we examined the behaviour of time series of measurements of horizontal wind speed and direction, temperature, potential temperature and ozone acquired from a suite of instruments on board the high altitude research aircraft M55 Geophysica during some of its missions to study the tropical upper troposphere and lower stratosphere.
We have selected for our analysis, horizontal (i.e. constant pressure) straight segments of all flights of the M55 aircraft satisfying the criterion of having duration greater than 3000s: given the aircraft's speed, the time series thus cover horizontal scales from 200 m to some thousands of km.
A synopsis of the dataset is reported in table 1:

flight_date	theta (K)	theta variability (K)	latitude (°N)	latitude range (°N)	mean temperature (°C)	duration (s)
_050120	425	5	4.3	-4,2/13	-54.1	4000
_050123a	407	10	28.5	21.9/35.2	-67.8	10000
_050123b	409	4	5.8	-2.7/12.7	-78	10000
_050127	386	2	-12.3	-15/.9.6	-83.6	6000
_050224_1	392	2	-17.5	-19.9/-14.9	-77.7	4900
_050224_2	350	2	-12.8	-14/-11.6	-64.3	3100
_050227_1	400	6	4.3	-4.3/13	-81.4	11000
_051104a	413	1	40.5	38/43	-67.4	4500
_051109	395	3	15.4	14.1/16.6	-82	6300
_051111	406	4	8	6/9.9	-79.2	4000
_051112	384	3	-3.8	-11/3	-82.5	11500
_051209	386	2	-4.6	-11/1.8	-81.3	10700
_051210_3	428	2	11.7	10.6/12.5	-77.1	3000
_051214	393	8	21.2	17.5/24.3	-76.5	12000
_051216_1	403	2	27.2	25.5/28.9	-72.4	6000
_060731_2	453	2	38.8	35.7/41.7	-62.6	5000
_060816_	448	8	17.6	14.4/20.8	-73.3	4000
_060817_1	437	2	35.6	33.1/38.3	-61.4	3700
_060817_2	448	3	41.5	39/43.9	-56.2	4500

Table.1: Flight tracks over great circle segments, at constant pressure.

For each variable we have computed the multifractal indices H_1, C_1, α whose use has been reported extensively in the literature (see, e.g. Pecknold et al., 1993; Davis et al., 1994; Seuront et al., 1999). For the same set of flight legs we have also computed the H_1 index for time series of concentration of condensation nuclei, nitrogen oxides, total water substance and carbon dioxide. We limited ourselves to only the more robust one of the multifractal indices due to the gappy nature of the time series for these latter variables, due to the need of instrument self-calibraton during the flights. Aim of the work was to contrast the exponents of variables that are known to behave as passive scalars with those that might undergo physical-chemical processes of loss and gain in the sampled airmasses.

We remind that H_q is the scaling exponent of the q - order structure function, defined as:

$$S_q(r;f) = \left\langle \left| f(t+r) - f(t) \right|^q \right\rangle$$

If *f* is a scale-invariant field, i.e. if *f* is such that a large scale structure is replicated at smaller scales, by plotting log $S_q(r;f)$ versus log *r*, we get a line whose slope $\zeta(q)$ defines the scaling exponent for *f*, and we define a non decreasing function as $H_q = \zeta(q)/q$.
It may be the case that H_q is not constant as q changes, then the field is called "multifractal", but it can be demonstrated that H_1 is a good scaling exponent for both the mono and multi fractal cases. H_1 ranges from 0 to 1 and is a measure of the nonstationarity (or "persistence", as correlation between adjacent values) of the measured field, with values close to 0 for rough, stationary signals, and values close to 1 for smooth, nonstationary signals.

If we define the quantities

$$\varepsilon(1,t) = \frac{\left| f(t+1) - f(t) \right|}{\left\langle \left| f(t+1) - f(t) \right| \right\rangle}$$

$$\varepsilon(r,t) = \frac{1}{r} \cdot \sum_{j=t}^{t+r-1} \varepsilon(1,j)$$

we note that, as r increases, the $\varepsilon(r,t)$ represent a more and more coarse-graining version of the original signal.
It turns out that for many fields $\langle \varepsilon(r,t)^q \rangle$ scales as $r^{-K(q)}$. If we define the non-decreasing function $C(q) = K(q)/(q-1)$, we can demonstrate that $C(1) = C_1$, that we will call intermittency parameter, quantifies the intermittency of the field, where C_1 is close to 0 for weakly variable fields, while is close to 1 for extreme δ–like, intermittency.

Finally α, also known as the Levy index, is linked to the steepness of the tails of the pdf of the field's variable and can vary between 0 and 2.

We have computed variogram for the geophysical fields under analysis, as in fig.1:

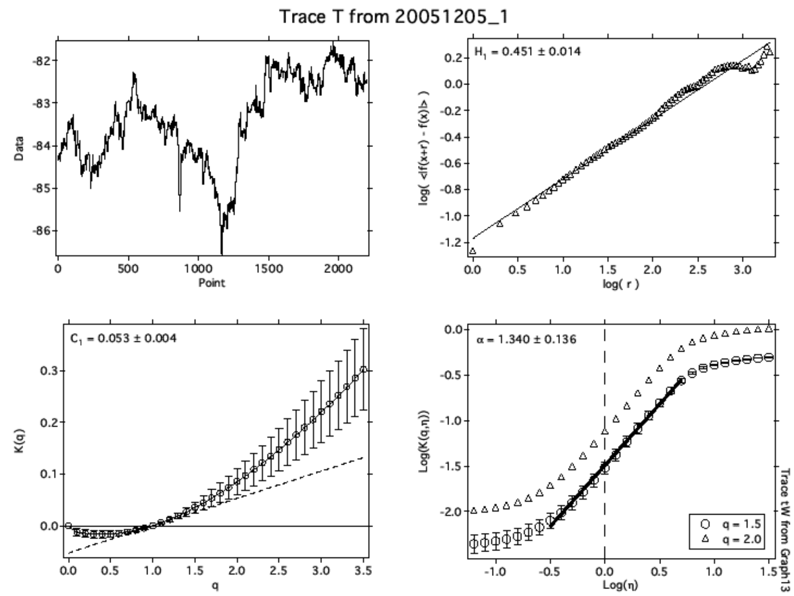

Fig. 1: upper left, temperature time serie, upper right variogram for the time serie. Lower left and right, computations for C1 and α.

A set of multifractal indices has been computed for the dataset under investigation.

We report the indices for wind speed and direction, temperature and ozone in the miltifractal planes (H,C) in fig. 2, (H, α) in fig. 3 and (C, α) in fig. 4.

The data are clearly multifractal and there is no significant pairwise correlation among them.

In fig. 5 the H indices for passive as well as active tracers are displayed vs potential temperature, while in fig. 6 the same indices are displayed vs distance, expressed in potential tempertature coordinates, from the cold point tropopause. The same H indices are also displayed in fig. 7 vs wind variability along the corresponding flight legs, used as a proxy for wind shear. While no significant trend can be discerned fo passive tracers, CCN, Ozone water seem to have their indices increasing with the wind shear. Finally fig. 8 shows the multifractal exponent C for temperature, vs temperature . The STM has allowed us to lay te foundations of a scale invariance analysis of active and passive tracers measured in the tropical tropopause region. A thoughtful interpretation of the retrieved values of the multifractal indices in terms of meteorological fields an physico-chemical processes will be the subject of future work.

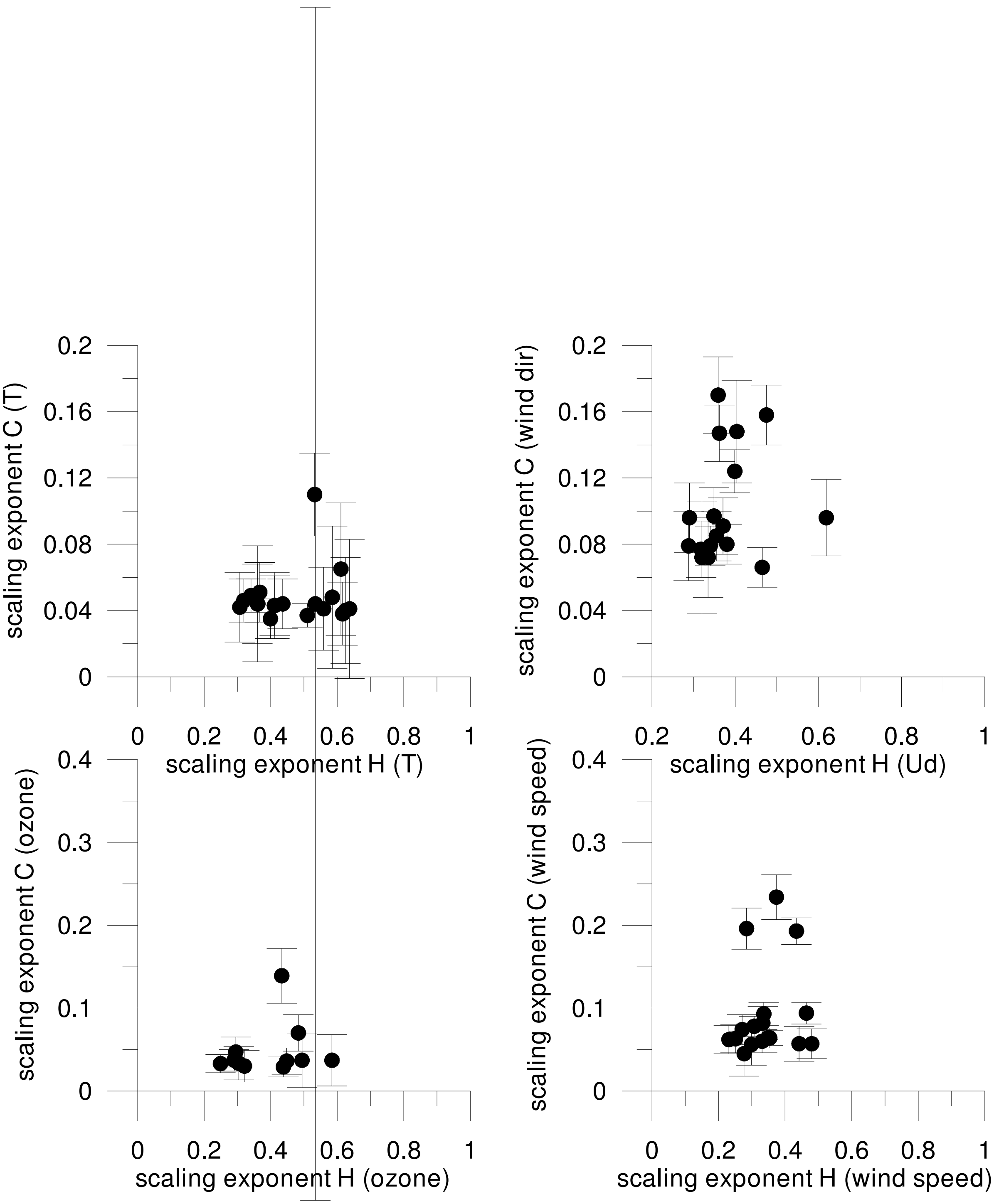

Fig. 1 Generalized scale invariance analysis of wind speed and direction, temperature and Ozone, in the (C, H) plane.

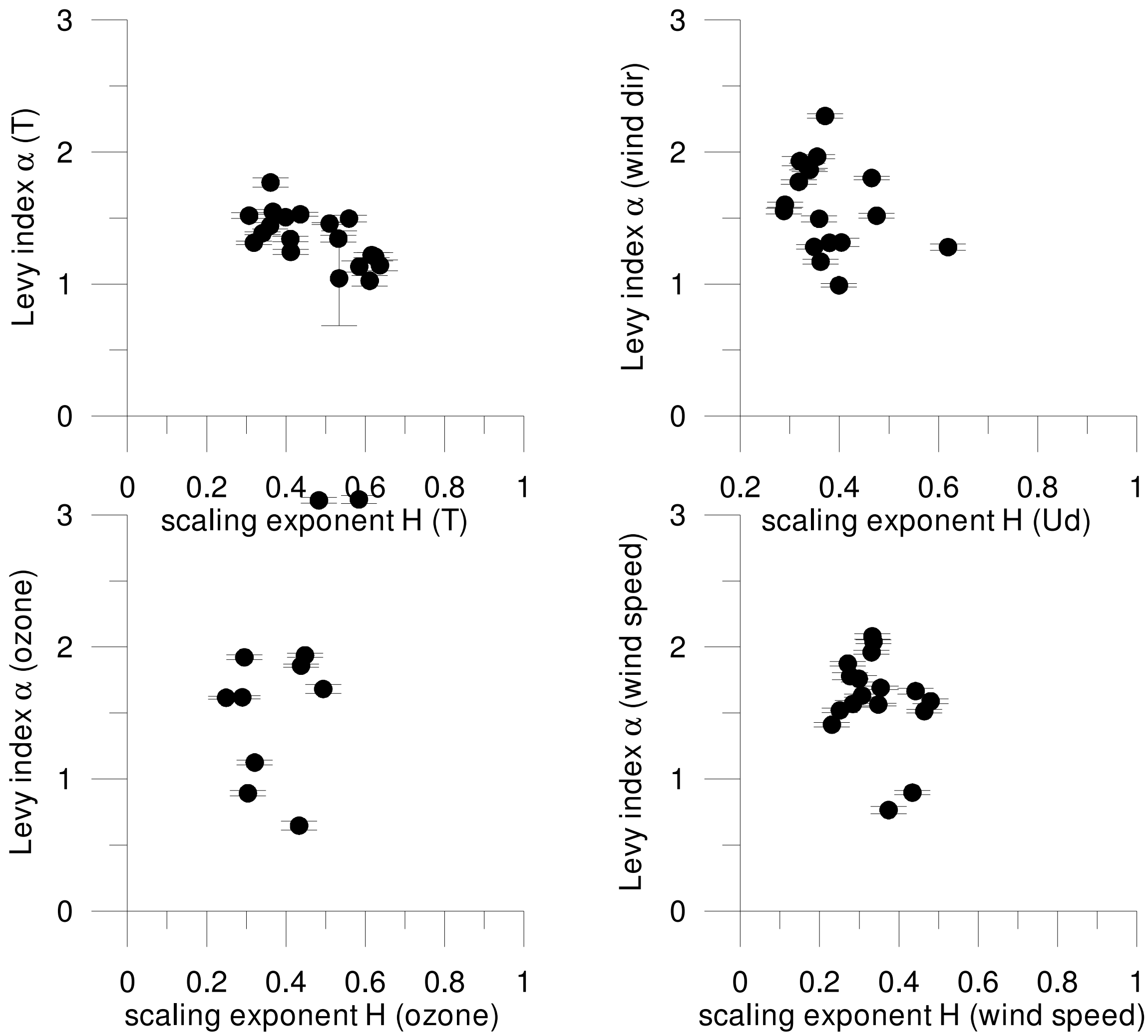

Fig. 2 Generalized scale invariance analysis of wind speed and direction, temperature and Ozone, in the (H, α) plane.

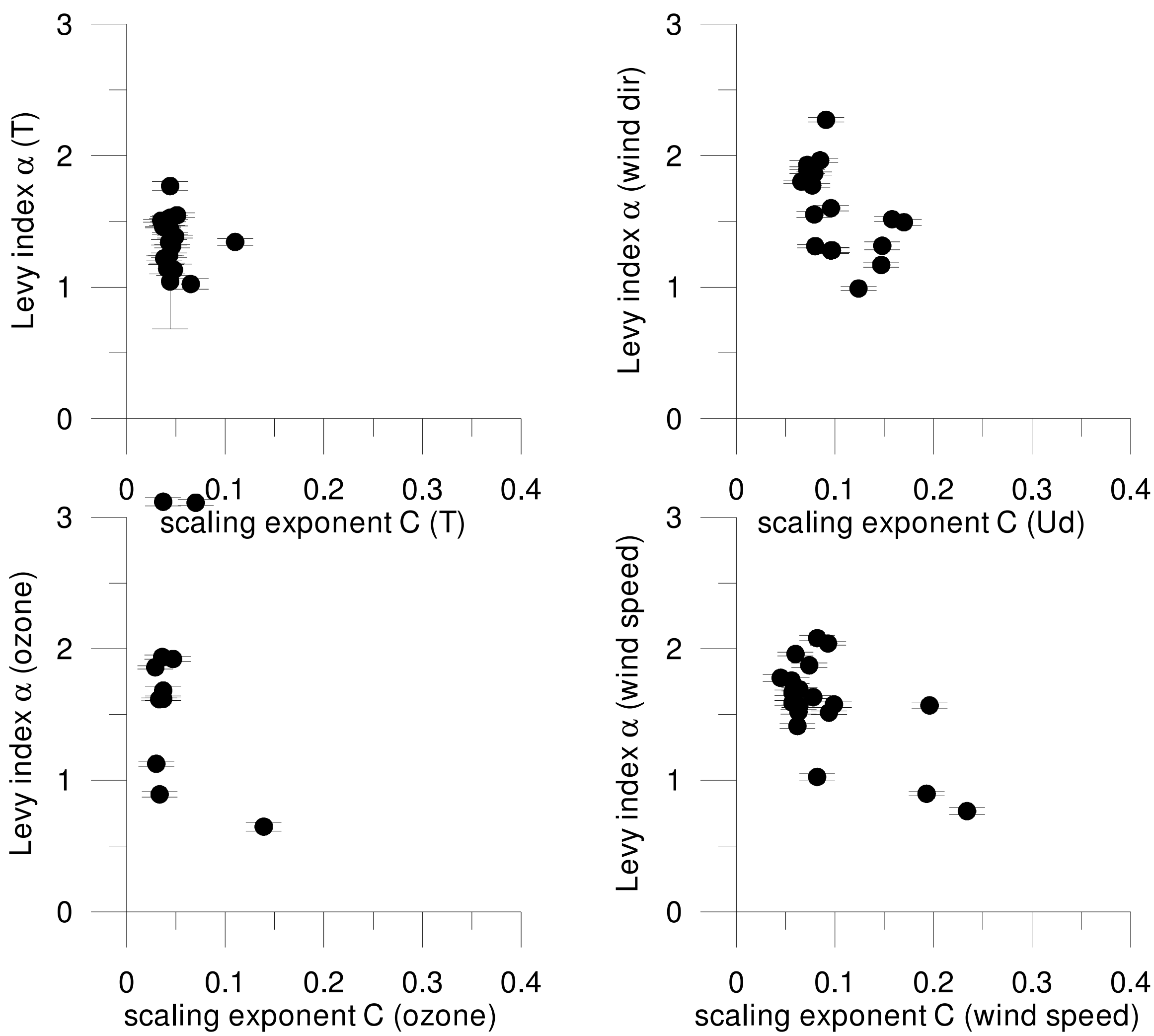

Fig. 3 Generalized scale invariance analysis of wind speed and direction, temperature and Ozone, in the (C, α) plane.

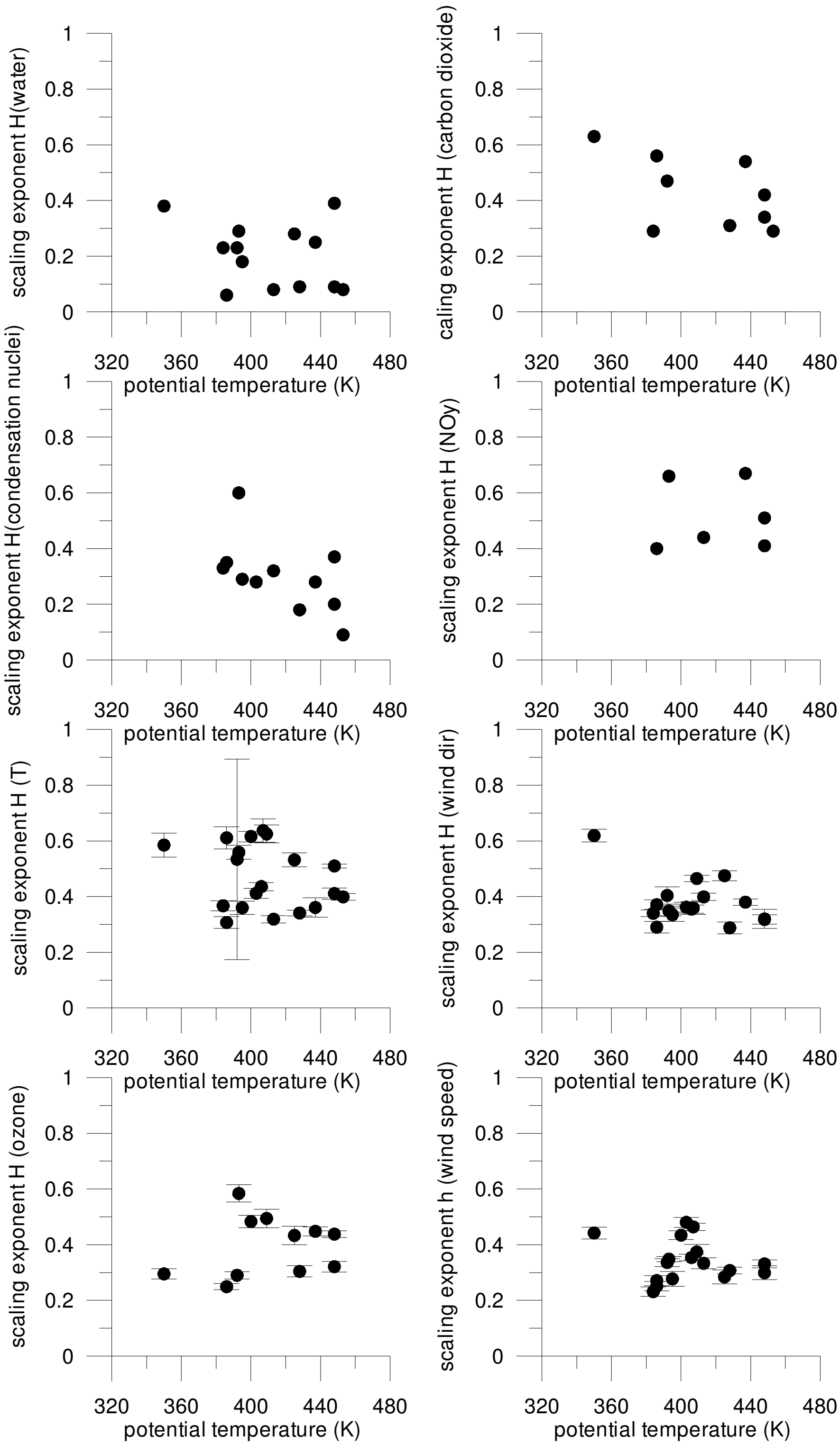

Fig. 5: H indices for water, NOy, CO2, CCN as well as for temperature, wind speed and direction and ozone, vs potential temperature.

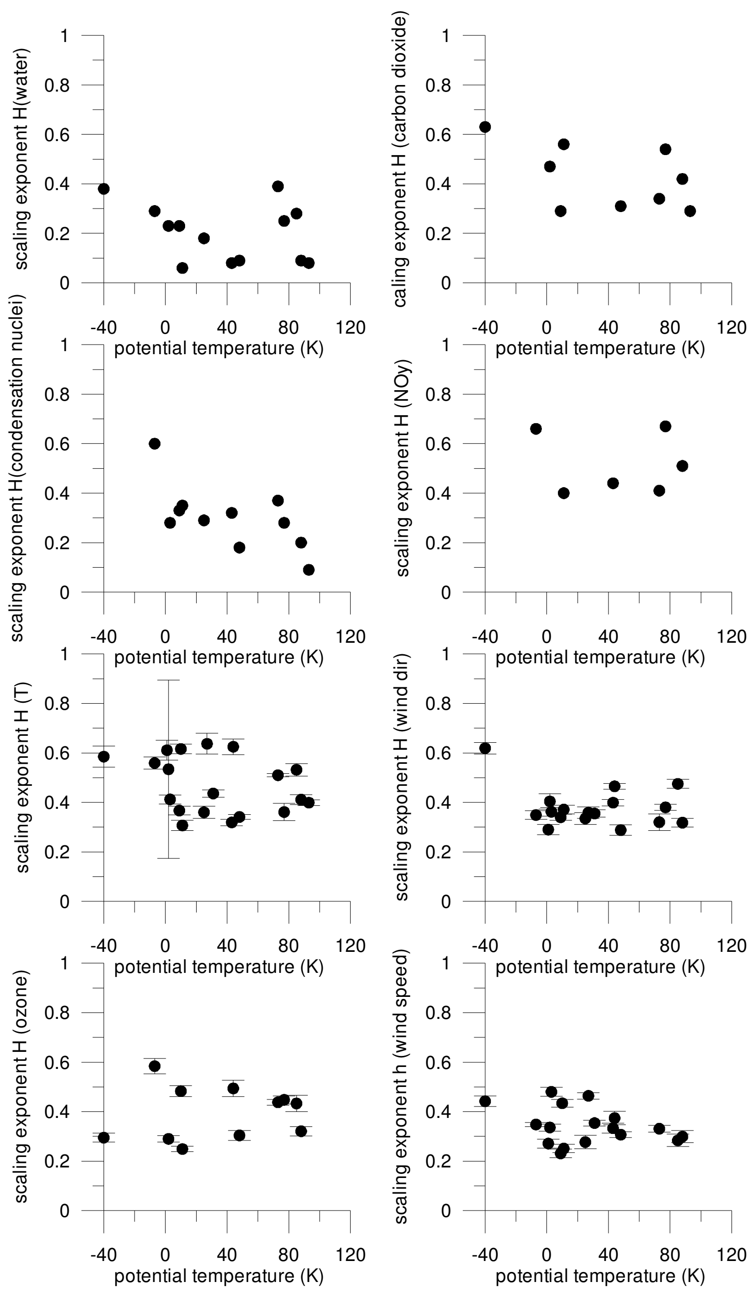

Fig. 6: H indices for water, NOy, CO2, CCN as well as for temperature, wind speed and direction and ozone, vs distance from the cold oint tropopause, expressed in potential temperature vertical coordinate..

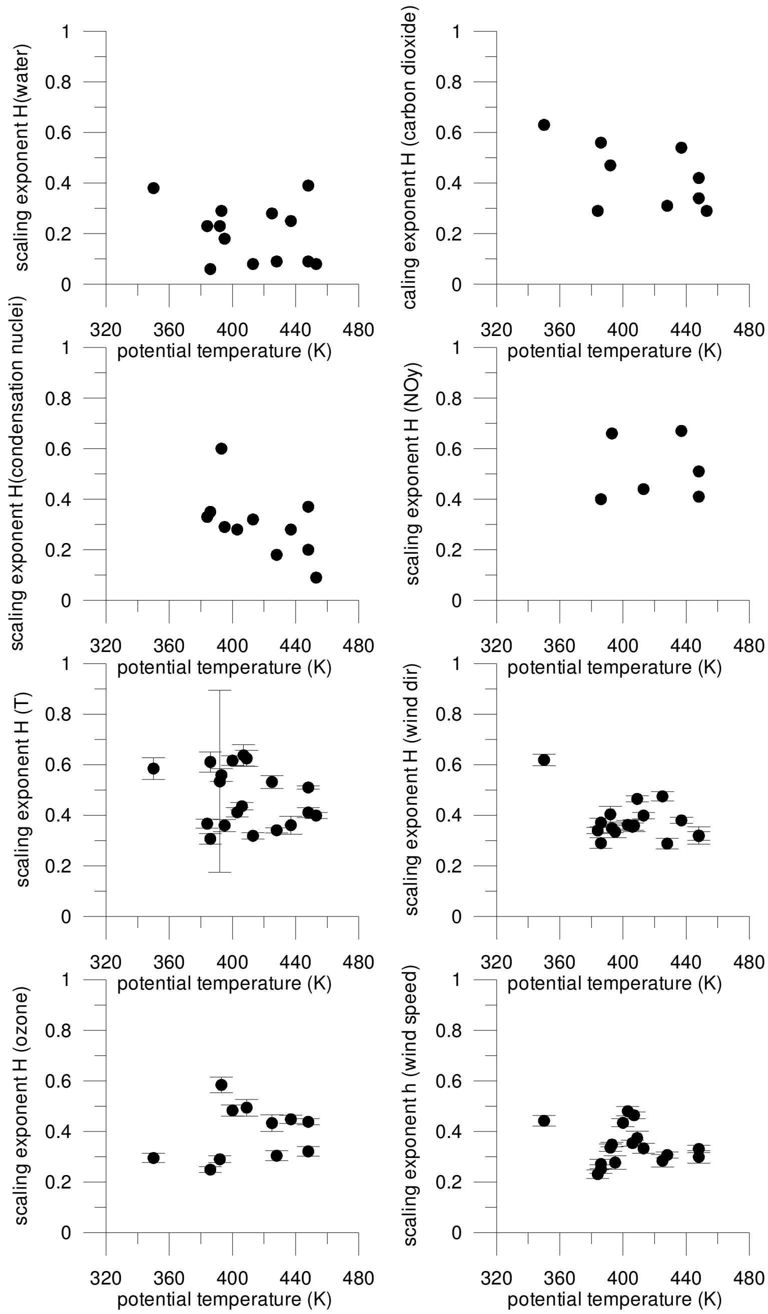

Fig. 7: H indices for water, NOy, CO2, CCN as well as for temperature, wind speed and direction and ozone, vs wind variability along the flight track.

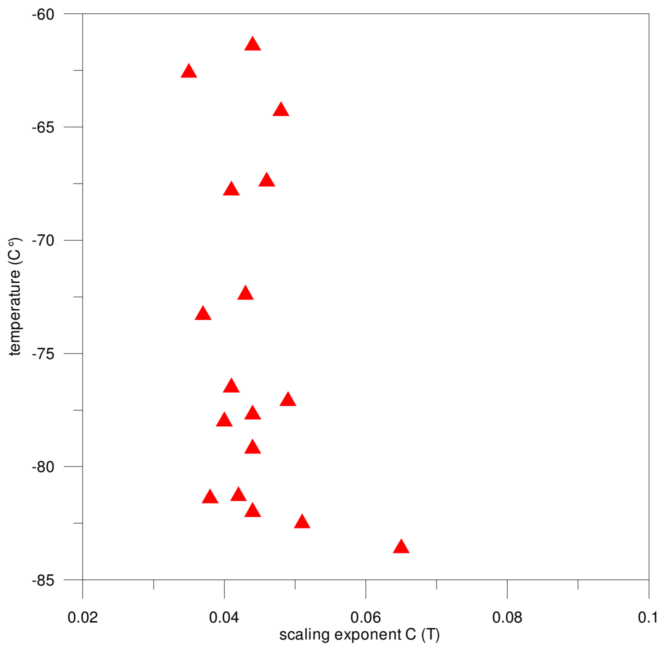

Fig. 8: Scaling exponent C for temperature, vs temperature.