Consiglio Nazionale delle Ricerche

PROGRAMMA SHORT – TERM MOBILITY – ANNO 2006

Relazione Finale del Programma:

Development of Fractional Diffusion models for the phenomenological description of observed atmospheric data.

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Dipartimento: Terra e Ambiente **Numero di codice: 1**

Introduction

In this programme a first attempt was made to study the possibility of applying the theory of Anomalous Diffusion (AD) [1] for the description of atmospheric transport phenomena and the development of the related models and equations.

AD phenomena are Non-Gaussian diffusion processes, arising as exceptions to the Central Limit Theorem. Their basic features are given by the power-law behaviour of some statistical indicators, such as the correlation function and the variance growth. In particular, the scaling of the variance (and in general that of the probability distribution) is not given by the Normal Diffusion scaling:

$$\xi^2 \sim \tau \qquad (1)$$

Power-law behaviour and anomalous scaling were already found in the literature, such as the power-law relation between the life-time of some atmospheric structures and their associated kinetic energy. This opens the possibility that some atmospheric trasport phenomenology could be described by AD models.

One basic approach to AD is essentially based on Renewal Theory (RT) [2], Continuous Time Random Walk (CTRW) [3] and Fractional Calculus (FC) [4]. In this approach a crucial physical mechanicm is the production of so-called "*Critical Events*" from the dynamics of the physical system. A theoretical example is given by the "turbulent" bursts generated by some dynamical maps, with short duration and high intensity, alternating with long period of "laminar" motion with low intensity of the signal.

RT is the branch of Probability Theory studying stochastic processes defined as sequences of "*Events*", in which the "*Waiting Times*" between two such events are independent random variables. The diffusion process is described by a CTRW model, i.e., a Random Walker where the diffusion is generated by the critical events. The distribution of Waiting Times is the first basic ingredient in the CTRW modelling, being the jump probability distribution the second one. Anyway, a long-range behaviour (i.e., large fluctuations) of the Waiting Times is enough to generate Anomalous Diffusion, even if the jump probability distribution has a finite variance and, due to the Central Limit Theorem, falls in the basin of attraction of the Gaussian Law (Normal Diffusion). If the jump probability distribution is chosen in this way, the scaling of the diffusion process is directly related to the power-law index of the distribution of Waiting Times.
Finally, the long-time behaviour of AD processes is described by Fractional Diffusion Equations, i.e., Diffusion Equation with Fractional Derivative Operators (derivatives of non-integer order).

An example is the Poisson process, which is a well known process satisfying the renewal condition, but it is related to Normal Diffusion, thereby the long-time behaviour is described by the standard Diffusion Equation. On the contrary, Non-Poisson renewal processes can generate Anomalous Diffusion. Essentially, this last kind of Non-Poisson renewal processes display power-law behaviour in the distribution of Waiting Times.

Consequently, the fundamental property that must be investigated is the distribution and correlation properties of "*Waiting Times*" between two "*Events*", where the main open questions are related to a good definition of "*Event*" in the atmospheric processes. Several definition are possible and most of them are not equivalent. For a given physical quantity given by atmospheric observations, such as the wind velocity or the temperature, the definition of event also depend on the temporal and spatial scales under consideration.
Following this approach, a first rough attempt of defining events was made and the distribution of Waiting Times was analyzed.

The statistical analysis on $PM_{2.5}$ concentration data

The statistical analysis performed here was focused on the problem of aerosol transport, generated by the advection of the atmospheric wind velocity field.
The statistical analysis was limited to atmospheric data at the ground level [5] and the typical time scale of turbulence was considered, trying to understand if the dynamics of aerosol particles can be described in the framework of AD. Time series of $PM_{2.5}$ concentration data were available. The data sampling of aerosol concentration was made with a frequency of 1 Hz, so that the micro-scale of turbulence is averaged out by the physical limitations of the instrumental apparatus performing the measure (optical sampler with frequency 1 Hz). Anyway, the main, and probably more important, portion of the turbulent spectrum is included in the measured time series, so that wind turbulence should affects the aerosol dynamics.

In Fig. 1 a sample of $PM_{2.5}$ concentration data is plotted, showing the typical variations related to the diurnal cycle. As a consequence, the time series is non-stationary and its statistical properties change with time. The statistical analysis was then performed on portions of the time series which can be considered almost stationary. An indicator of quasi-stationarity is given by the requirement that the Monin-Obhukov length, determining the atmospheric stability in the Planetary Boundary Layer, is nearly constant or, at least, it does not change sign.
In Fig. 2 an example of the quasi-stationary samples, considered for the analysis, has been reported, including approximately 4 hours of observations in a day-light period far from sunrise and sunset, where the non-stationary effects are much stronger.

The first attempt of defining events was made by simply imposing a threshold C. Then, an event is generated when the signal increases over the threshold and the Waiting Times between such

events are computed.
In Figs. 3-4 the sequences of Waiting Times τ_κ for two different thresholds are displayed and in Figs. 5-6 the relative survival probability distribution are reported. The survival probability is defined as the probability of having Waiting Times larger that a given value:

$$\Psi(\tau) = \{\tau_\kappa > \tau\} \quad (2)$$

From Fig. 5-6 it is possible to see that the distribution of Waiting Times display some kind of power-law behaviour with crossovers. Such crossovers have some dependence on the threshold.
The plot in Fig. 5, given by the threshold C=37.5, shows an apparent crossover from an exponential decay in the short-time region to a power-law decay in the relatively long-time behaviour, while in Fig. 6 the crossover is between two different power-law decay. Anyway, these analyses deserve some more effort in order to increase the statistics (number of events), that here in quite poor, and the time range, being the power-law decays restricted to only one decade.
An interesting point is that, in the long-time range, power-law decay seems to have the same power-law index. This could be a signature of some kind of "universal" behaviour, related to the internal dynamics of aerosol transport (probably affected by air turbulent motions) and independent by external (non-stationary) forcings.

A first conclusion is that aerosol dynamics diplays some features of Anomalous Diffusion.
On the contrary, the applicability of RT, CTRW and FC is not well established. As described above, a basic requirement is that the Waiting Times must be uncorrelated. This was checked by using a novel technique of time series analysis [6], based on the "aging" properties of renewal processes, which is indicated as "*Renewal Aging*". This analysis did not give clear results and the applicability of RT remains an open question. Then, further investigations on the statistical features of these time series are required. In particular, a crucial point is the need of defining "Events" in a proper way.
Also some theoretical effort on CTRW and FC modeling and on the possible extension of such models is needed.

Bibliography

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[6] P. Allegrini, F. Barbi, P. Grigolini, **P. Paradisi**: *Renewal, modulation, and superstatistics in times series*, Physical Review E **73** (4), 046136 (2006) .

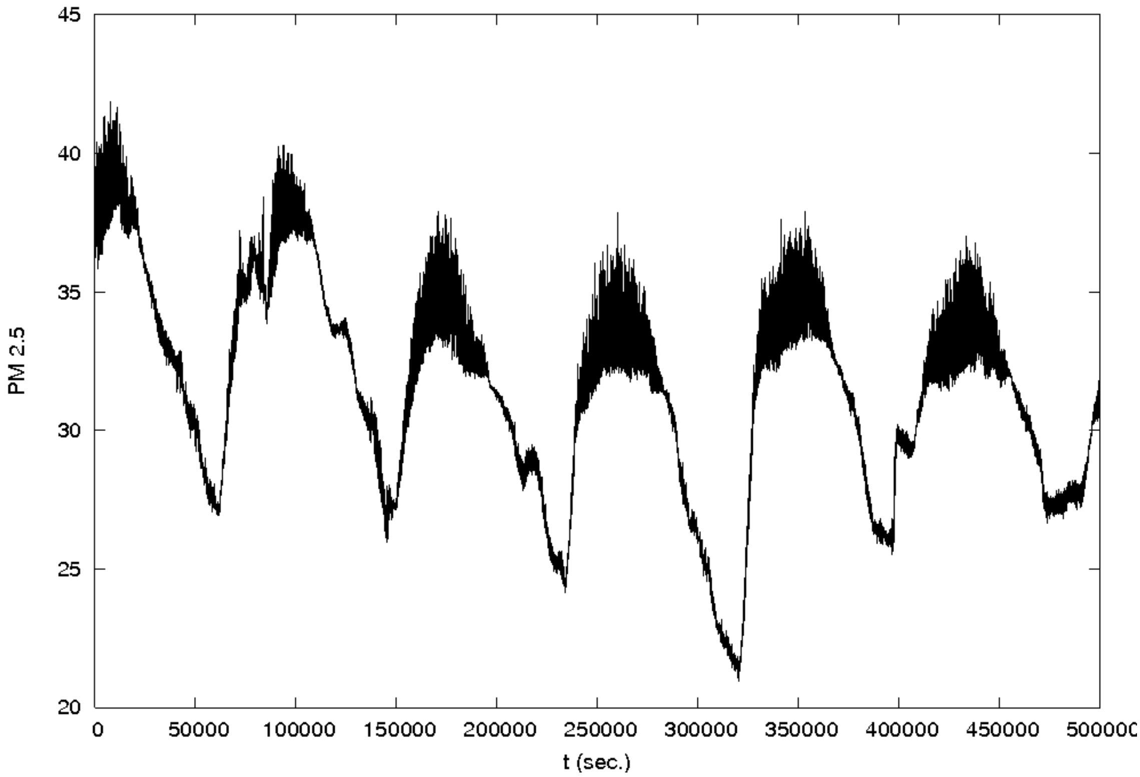

Figure 1: A sample of $PM_{2.5}$ concentration time evolution, showing the typical diurnal variations.

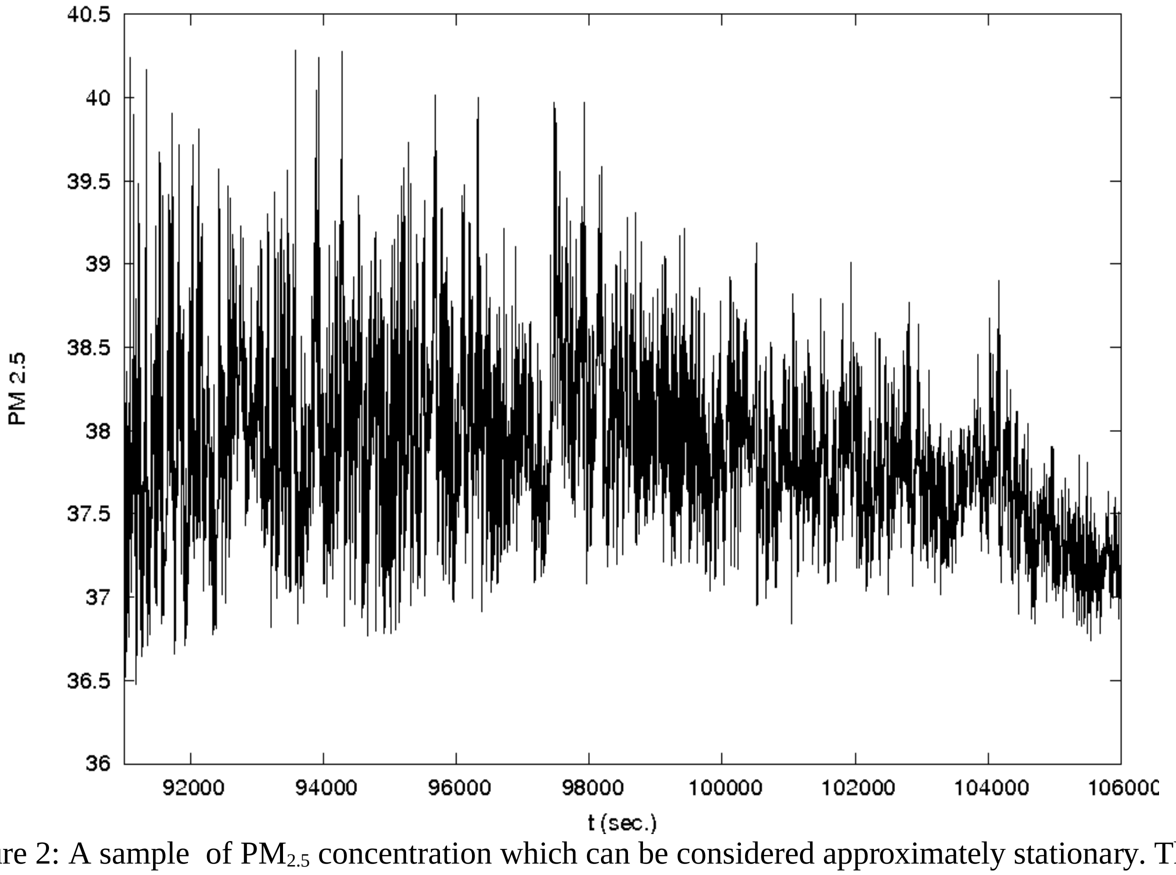

Figure 2: A sample of $PM_{2.5}$ concentration which can be considered approximately stationary. The sample includes about 4 hours of observations.

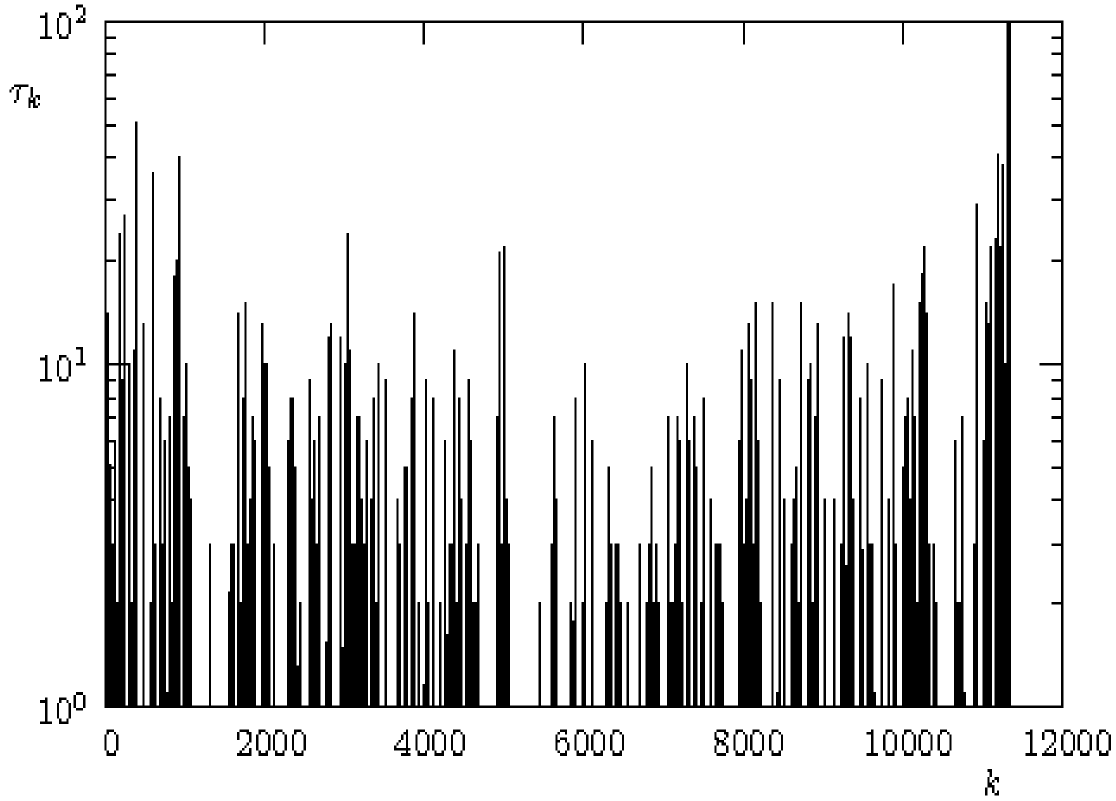

Figure 3: Sequence of Waiting Times related to the events generated by the threshold C=37.5.

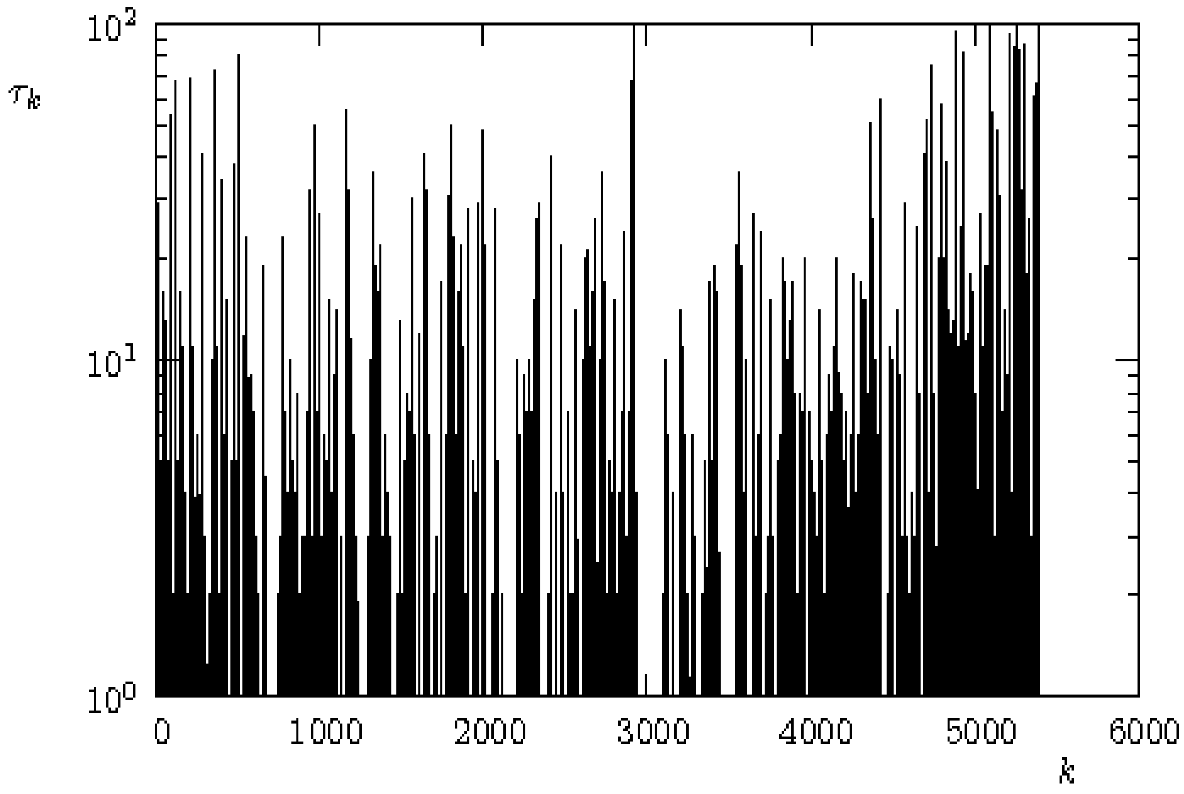

Figure 4: Sequence of Waiting Times related to the events generated by the threshold C=38.

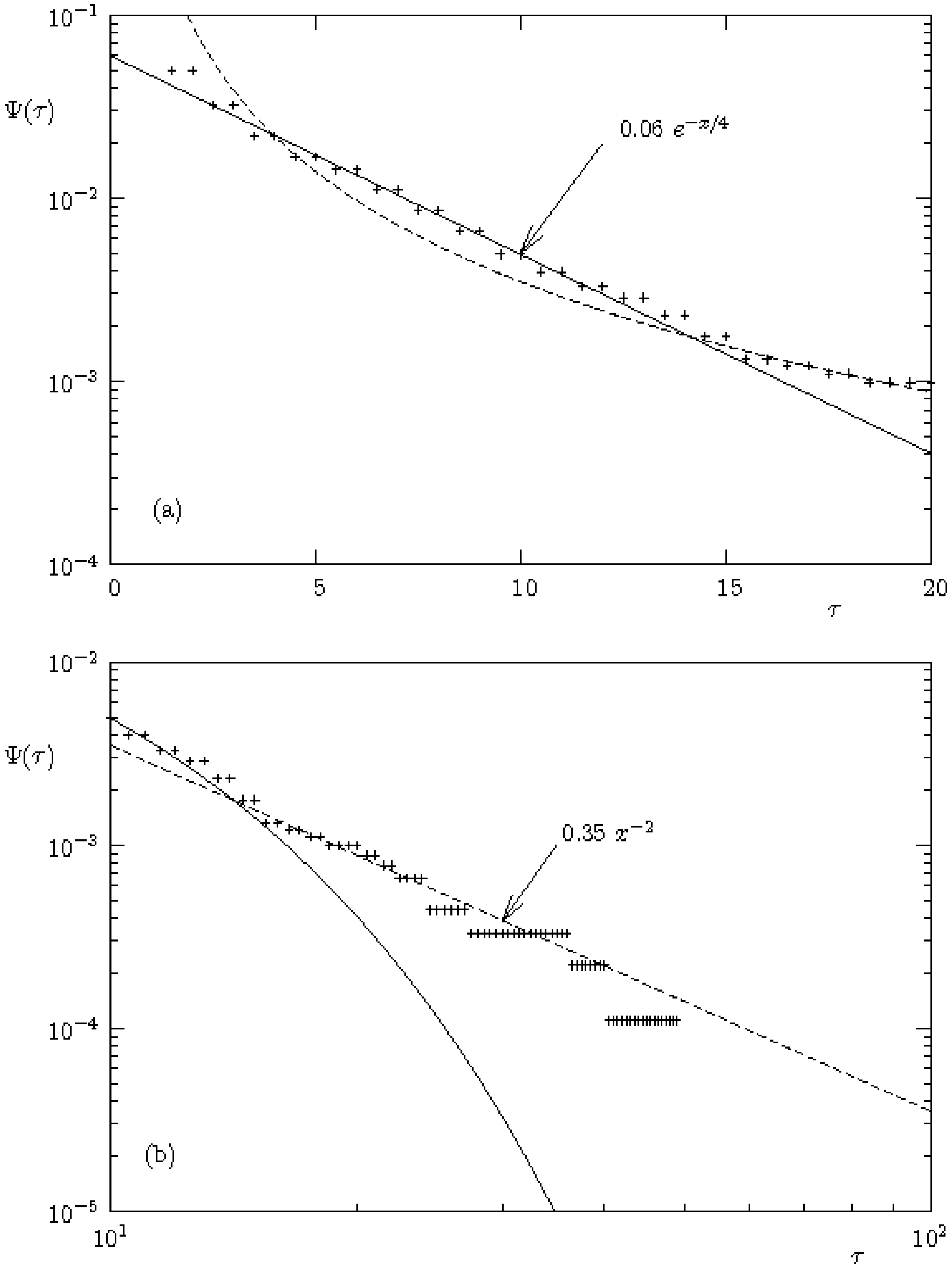

Figure 5: Survival probability of Waiting Times. Threshold C=37.5

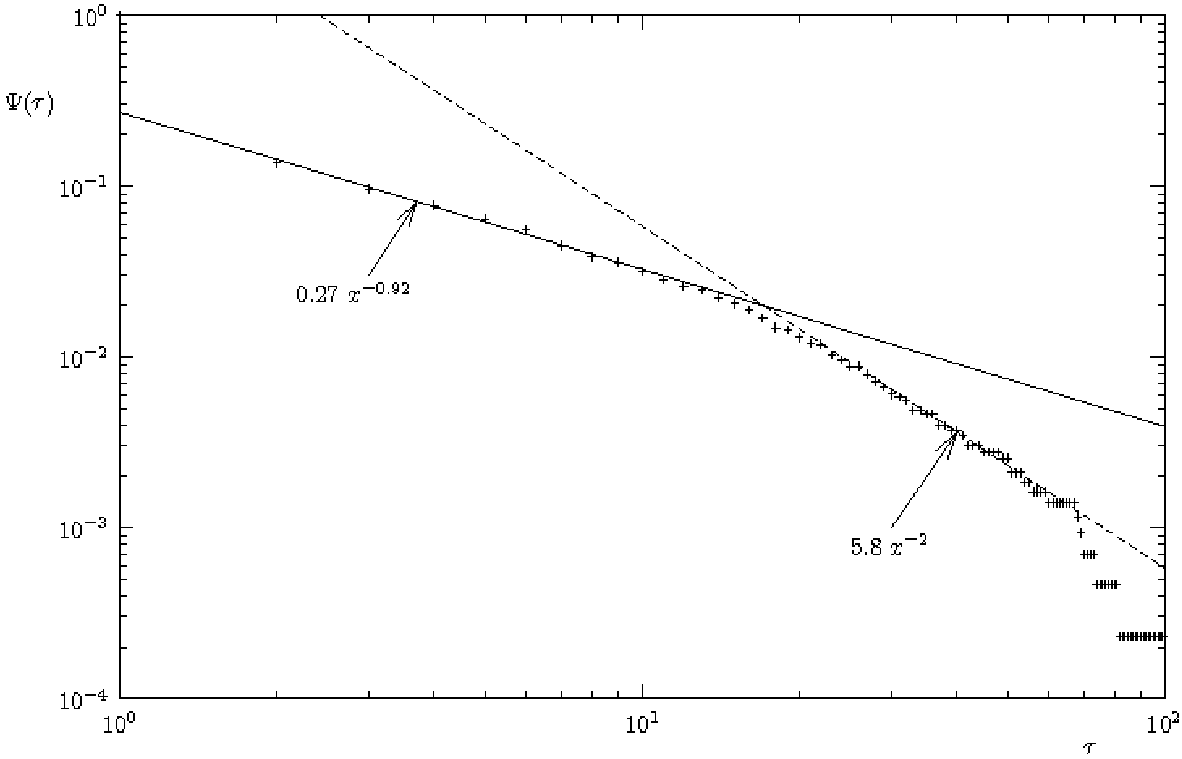

Figure 6: Survival probability of the Waiting Times. Threshold C=38.

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